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Mathematically Motivating Students with Photography and GeoGebra while Addressing Math Anxiety

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Abstract

The paper discusses about the importance of using technology in the teaching of elementary mathematics, particularly, GeoGebra. GeoGebra is an emerging technology in Florida, the USA, and around the world. GeoGebra, a dynamic mathematics software, can assist in developing a deeper understanding of geometric/measurement/algebraic concepts in the mathematics classrooms from Grades K-16. Several GeoGebra activities appropriate for the elementary levels are shared in this paper. The easily accessible files can be downloaded freely by teachers to be used in their classrooms. Emphasis on addressing math anxiety and using photography to teach math using the GeoGebra software are the main foci of the paper.

Keywords: Common Core State Standards; GeoGebra; Geometry; Math Anxiety; Mathematics; Photography.

*"Of all of our inventions for mass communication, pictures still
*****speak the most universally understood language."*

*****-- Walt Disney Company

Introduction

The quote above is touching and so true, our young children in our math classrooms today can learn so much more through pictures and photography. This paper will touch on the need to turn students on to math for a STEM World. It will look at how by using photography and GeoGebra, we can better reach our students and show them how math surrounds us. In today's technologically oriented world, students need to be proficient in Science, Technology, Engineering, and Mathematics (STEM) fields. As endorsed by the National Council of Teachers of Mathematics (NCTM, 2000) and stressed in the new Common Core State Standards in Mathematics, it is critical that we teach using technology, address attitudes and anxiety toward math, and make the math students are learning relevant and meaningful to the learners. Often, it may be best to start teaching young people geometry first as opposed to numbers as numbers are considered to be more abstract and difficult to learn initially. Geometry is one of the most concrete branches of mathematics and focusing on this first can benefit students' whole view of mathematics and their attitudes towards learning it. Today teachers also need to be cognizant and be checking for attitudes and dispositions toward learning mathematics as math anxiety is a real issue in today's classrooms. This paper looks at ideas for teaching mathematics with the use of technology and photography using the free dynamic mathematics software GeoGebra, that will help teachers develop in their students a variety of expertise described in the Standards for Mathematical Practices numbers 4 and 5: model with mathematics, and using appropriate tools strategically.

Math Anxiety: The Need to Check for Mathematical Dispositions

Math anxiety is defined as feeling of anxiety that one cannot perform efficiently in situations that involve the use of mathematics. Although it is mostly associated with academics, it can apply to other aspects of life (eHow Website, n.d.) Richardson and Suinn (1972) originally defined math anxiety as "a feeling of tension and anxiety that interferes with the manipulation of numbers and the solving of mathematical problems in a wide variety of ordinary life and academic situations" (p. 551). Mathematics anxiety is the "irrational dread of mathematics that interferes with manipulating numbers and solving mathematical problems within a variety of everyday life and academic situations" (Buckley and Ribordy, 1982, p. 1).

Math anxiety research has been around since the 1970's (Richardson & Suinn, 1972). Math anxiety still continues to plague our society and affects our young peoples' success and achievement with the subject (Quander, 2013). Quander



feels that elementary teachers need to help prepare students to be lifelong learners and develop a productive mathematical disposition so that they are prepared for future schooling and eventual careers. Math anxiety can impede not only mathematical performance but also interest and then career choice and many decisions in life. Beilock and Willingham (2014) feel that math anxiety robs people of working memory, which is important for solving problems and doing math and that math teachers can do more in classrooms to help reduce math anxiety for their students. The idea of looking closely at math anxiety levels, motivation to learn mathematics, and using technology like GeoGebra to teach and motivate students is critical today in a world of STEM.

According to NCTM (1989), mathematics teachers need to assess students' mathematical disposition regarding:

- confidence in using math to solve problems, communicate ideas, and reason
- flexibility in exploring mathematical ideas and trying a variety of methods when solving problems
- willingness to persevere in mathematical tasks
- interests, curiosity, and inventiveness in doing math
- student ability to reflect and monitor their own thinking and performance while doing math
- value and appreciate math for its real-life application, connections to other disciplines and cultures and as a tool and language

That only 7 percent of the population in their study reported having positive experiences with mathematics from kindergarten through college. Their study cited that there are many covert (veiled or implied) and overt (apparent and definite) behaviors exhibited by the math instructor in creating math anxiety in students. Things like difficulty of material, hostile instructor behavior, gender bias, perceptions of uncaring teacher, angry behavior, unrealistic expectations, embarrassing students in front of peers, communication and language barriers, quality of instruction, and evaluation methods of the teacher. Math instructors' behaviors and teaching methods can be hurtful and negative to students learning math. "I like the class because of the teacher." It is often because the teacher knows how to present developmentally the subject matter, creates a learning environment conducive to learning with compassion, has high expectations for all students without regard to gender, race, or language barriers, and uses a variety of assessment methods and teaching styles to better reach all students.

How to prevent math anxiety:

1. Using "Best Practice" in mathematics such as: manipulatives, cooperative groups, discussion of math, questioning and making conjectures, justification of thinking, writing about math, problem-solving approach to instruction, content integration, technology, assessment as an integral part of instruction, etc.
2. Incorporating the NCTM and State/Common Core Math Standards into the curriculum and instruction.
3. Discussing feelings, attitudes, and appreciation of mathematics with students (Furner, 2007).

How to reduce math anxiety:

1. Psychological Techniques like anxiety management, desensitization, counseling, support groups, bibliotherapy, and discussions.
2. Once a student feels less fearful about math, he/she may build their confidence by taking more mathematics classes.
3. Most research shows that until a person with math anxiety has confronted this anxiety by some form of discussion/counseling no "best practices" in math will help to overcome this fear (Furner, 2007).

According to Zemelman, Daniels, and Hyde (2012), we need to use best practices in teaching math to make math instruction most effective, things such as:

- Use of manipulatives (concrete math)
- Cooperative group work
- Discussion of math
- Questioning and making conjectures
- Justification of thinking
- Writing in math: thinking, feelings, and problem solving
- Problem-solving approach to instruction
- Content integration and real-life application
- Use of calculators, computers, and all technology
- Being a facilitator of learning
- Assessing learning as a part of instruction

As teachers we need to do things like giving a math attitude survey or read the book *Math Curse* to get students to talk about true feelings toward math?

We need to teach math using the CRA Model for teaching as follows:

1. Start with the Concrete using hands-on manipulatives like Geoboards
2. Move to Representational models in diagrams (or use Virtual Manipulatives like NLVM at: <http://nlvm.usu.edu/>)
3. Lastly, connect to the Abstract symbolism where student understand and function at an abstract level completely (GeoGebra software works well at: <http://www.geogebra.org/cms/en/>)

Connections need to be made when we teach math. Munakata and Vaidya (2012) based on their research found that students do not consider mathematics and science to be creative endeavors, though the traditional artistic disciplines rank high in this regard. To address this problem in perception, the authors used photography as a means to encourage students to find the deep-rooted connections between science and mathematics and the arts. The photography project was

used in a formal classroom setting as well as an outside activity, i.e. in a more informal setting. The project found student interest and motivation were peaked when photography was part of the instructional strategies to teach new material while making meaningful connections to the math using the photography.

Common Core Math Standards we can teach using GeoGebra:

Grade 3

Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures. Reason with shapes and their attributes.

3.G.1. Understand that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories. See Figure 1.

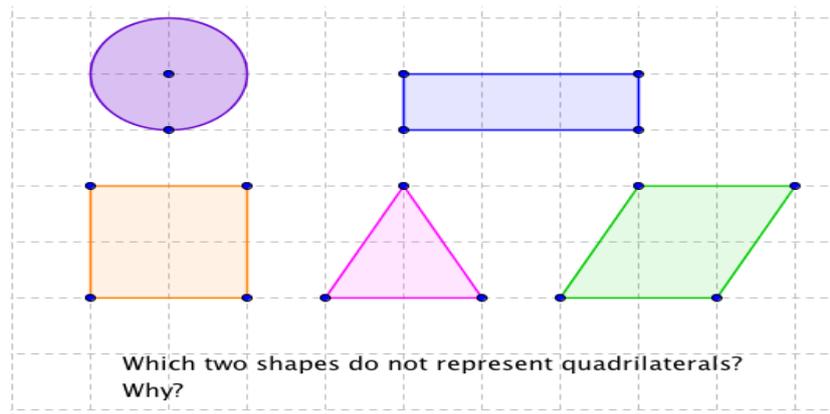
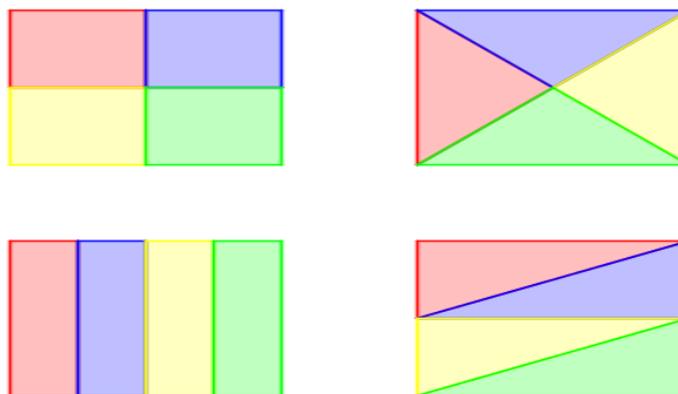


Figure 1

3.G.2. Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole. For example, partition a shape into 4 parts with equal area, and describe the area of each part as $\frac{1}{4}$ of the area of the shape. See Figure 2.



Use the area tool in GeoGebra to verify if the area of the partitions are $\frac{1}{4}$ of the area of the shape.

Figure 2

Grades 6

6.G.1. Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.

6.G.3. Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems. See Figure 3.

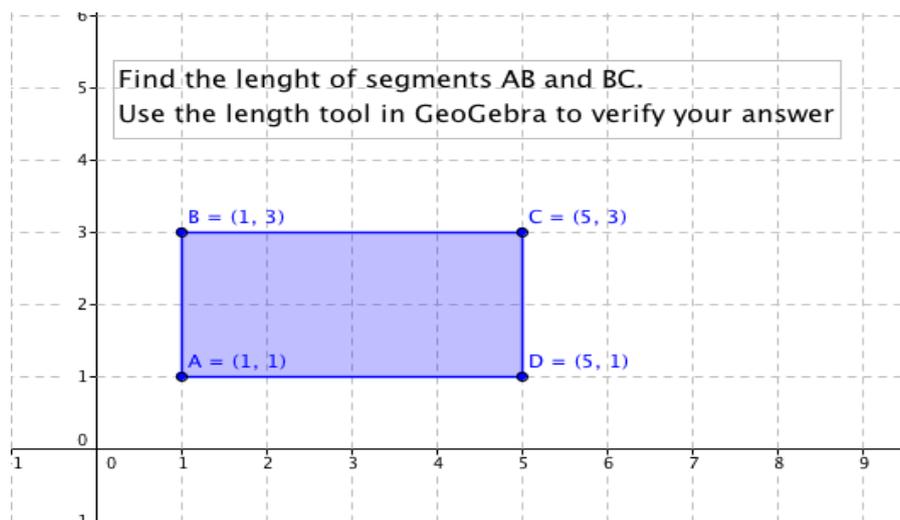


Figure 3

By relating and understanding real-world problems through the use of dynamic technology like GeoGebra and connecting them to photography to make important connections in math, our learners recognize that geometry and shapes/mathematics surround us!

See Appendix B for more ideas.

Technology use in the Teaching of Mathematics

The use of technological tools is critical in today's world. Our students need to learn to work at higher levels of generalization, model and solve complex problems, and focus on decision-making and reasoning (National Council of Teachers of Mathematics (NCTM) 1989, 2000, 2006). NCTM believes that mathematical power can arise from technology that includes: increased opportunity for learning, increased opportunities for real-life social contexts, and orientation to the future.

NCTM believes that technology in school mathematics refers to:

- digital tools
- computers
- calculators
- other handheld devices
- dynamic software
- podcasts
- interactive presentation devices
- spreadsheets
- Internet-based resources
- emerging technology and novel uses of technology

NCTM's focus in using technology is to: promote technology as an essential tool for learning mathematics in the 21st century, integrate the principles and process standards with teaching the content standards, provide access to all five mathematics content standards for all students. By providing learner-centered strategies that address the diverse needs of all learners of mathematics, NCTM feels that effective teachers maximize the potential of technology to: develop students' understanding, stimulate their interest, and increase their proficiency in mathematics.

The President's Council of Advisors on Science and Technology (PCAST) (Holdren, Lander, & Varmus, 2010) released an executive report in November 2010 where specific recommendations to the administration are given to ensure that the United States is a leader in Science, Technology, Engineering, and Mathematics (STEM) education in the coming decades. One of the recommendations is to recruit and train 100,000 new STEM middle and high school teachers over the next decade that are able to prepare and inspire students and have strong majors in STEM fields and strong content-specific pedagogical preparation. PCAST regards teachers as the most important factor in ensuring excellence in STEM education. Despite the ongoing efforts to promote the use of technology in education (e.g., National Council of Teachers of Mathematics [NCTM], 2000; National Educational Technology Standards for Teachers [NETS.T], 2008), teachers' ineffective use of technology has been reported in the literature. One reason frequently cited is that teachers are not trained in utilizing technology in the classroom within context. Hwang, Su, Huang, & Dong, (2009) found that by combining virtual manipulatives and software like GeoGebra along with white board, teachers can better model problems, help students understand and solve the problems while reaching higher levels in the teaching of many mathematical ideas in the curriculum.



Geogebra

GeoGebra is free and multi-platform dynamic mathematics software for all levels of education that joins geometry, algebra, tables, graphing, statistics and calculus in one easy-to-use package (Hohenwarter, Hohenwarter, & Lavicza, 2009). It can be downloaded for free and accessed at: <http://www.geogebra.org/cms/en/info>.

In research by Fahlberg-Stojanovska, & Stojanovski (2009) they found that using GeoGebra is motivating for students and helps them learn at a higher level while exploring and conjecturing as they draw and measure. Rosen & Hoffman (2009) found that it is very important to integrate both concrete and virtual manipulatives into the math classroom, such as representational models like GeoGebra. Furner & Marinas (2007) found that young people can easily transition from the concrete when using manipulatives like geoboards to the abstract when using geometry sketching software like GeoGebra. The Appendix A provides online websites on resources related to GeoGebra.

GeoGebra is an open-source dynamic mathematics software. Thus, because of its open-source nature there are no licensing issues associated with its use, allowing students and teachers freedom to use it both within the classroom and at home. Secondly, GeoGebra combines dynamic geometry, algebra, calculus, and spreadsheet features (which other packages treat separately) (Hewson, 2009) into a single easy-to-use package making it suitable for learning and teaching mathematics from elementary through university. Thirdly, GeoGebra has a large international user and developer community with users from 190 countries. The software is currently translated into 55 languages and attracts close to 300,000 downloads per month.

The most powerful feature of GeoGebra is the connection it makes between Geometry, Algebra, Calculus and Statistics. GeoGebra is a dynamic geometry system in which you work with points, vectors, segments, lines, and conic sections. GeoGebra is also a dynamic algebraic system, where equations and coordinates can be entered directly. Functions can be defined algebraically and then changed dynamically afterwards. GeoGebra has a simple CAS in the background, which has the ability to deal with variables for numbers, vectors and points, find derivatives and integrals of functions and offers commands like Root or Extremum. These two views are characteristic of GeoGebra: an expression in the algebra window corresponds to an object in the geometry window and vice versa. The spreadsheet view has been added recently making it possible to enter data in the spreadsheet and view graphs in the geometry window while maintaining its dynamic characteristic. Although GeoGebra has been primarily intended for mathematics instruction in secondary schools, it certainly has uses in Higher Education and even now being brought down to the elementary math levels as well.

Professional Training of Geogebra use and Teacher Resources

The need to make teachers proficient in the use of technology in the classroom is increasing rapidly. However, knowledge of the technology does not guarantee good use of the technology in the classroom. The question of what teachers need to know in order to incorporate technology into their teaching has received a great deal of attention in the last decade (International Society for Technology in Education, 2000; National Council of Teachers of Mathematics, 2000). Andresen & Misfeldt (2010) based on their research have found that with the new Common Core Standards (National Governors Association Center for Best Practices (NGA Center) and the Council of Chief State School Officers (CCSSO), 2010) in teaching mathematics that teachers need to be trained and also learn new math content and technology like GeoGebra in order to be effective in teaching and reaching the students that they will work with. Mishra & Koeler (2006) have introduced the term „Technological Pedagogical Content Knowledge“ in order to describe a framework for the teacher’s knowledge necessary to integrate technology in the classroom. Knowledge of technology cannot be isolated from the content, and good mathematics teaching requires an understanding on how technology is related to the pedagogy and mathematics (Hughes, 2005). In the institute, GeoGebra has been integrated in all courses interweaving all three key sources of knowledge: technology, pedagogy, and mathematics.

This paper will provide the following examples that illustrate some of the concepts presented in Grades 6 level and how GeoGebra can help math teachers teach and explore, discover, and understand some very abstract math concepts. See Figure 4.

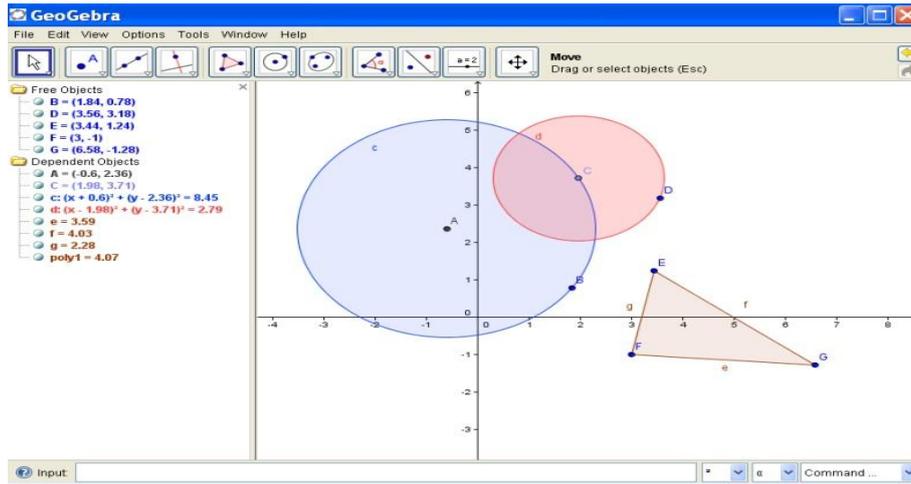


Figure 4

A brief overview of some of the Buttons and what they can do in GeoGebra. See Figure 5.

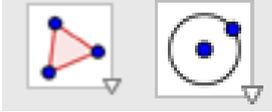
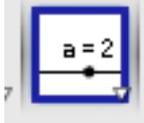
	<p>How to use the polygon and circle tools</p>
	<p>How GeoGebra uses sliders to adjust values</p>
	<p>How to use checkboxes to hide and show functions</p>
	<p>How to insert images into a file</p>

Figure 5

Some practice activities to learn how to use GeoGebra are:

- Open a blank GeoGebra file and show how to close the algebra view and how to display the grid and axes.
- Use the polygon tool to create a polygon and show how to find the length of a side of the polygon, the area and perimeter of the polygon. Then use the circle tool to show the area and circumference of a circle.
- Mention that the upcoming file on linear functions will show how sliders are used for various values of slope and y-intercept.
- In the slide on functions, we will show how checkboxes can be used to show multiple functions in one file.

GeoGebra can be used to show how mathematical equations can be applied to everyday objects. Aydin & Monaghan (2011) in their research feel that math teachers need to explore the potential for students to "see" mathematics in the real world through "marking" mathematical features of digital images using a dynamic geometry system like GeoGebra. Elementary math teachers may find the following videos (Mathematics and Multimedia, n.d.) of basic training for GeoGebra at: <http://mathandmultimedia.com/2011/01/01/geogebra-essentials-series/> useful as these videos provide a great resources for how to quickly use GeoGebra and classroom teachers may find the links and online training beneficial to easily use GeoGebra with their students while teaching.

Besides GeoGebra, there are many other free online math teaching tools (See Appendix A). The Virtual Websites provide great representational understanding for students when learning math concepts at:

- National Library of Virtual Manipulatives (See Figure 6)
- National Council of Teachers of Mathematics
- Cut the Knot
- Other Resources

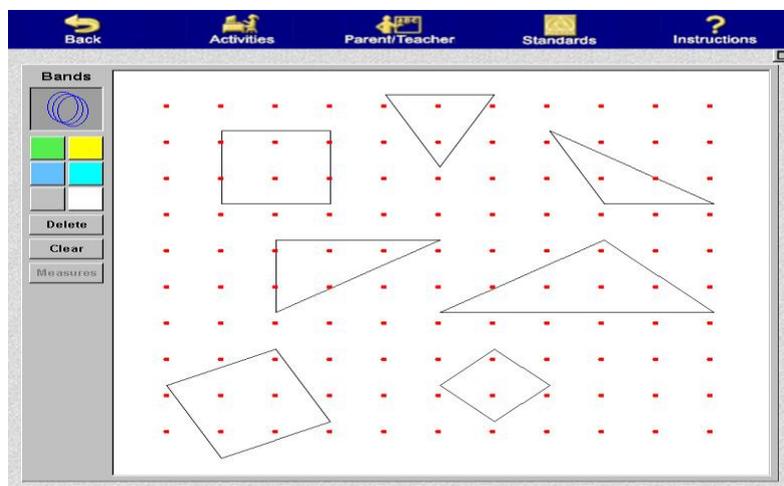


Figure 6

Common Core Standards as they Relate to using Geogebra

Today, most schools and states are adhering to the new Common Core Math Standards (National Governors Association Center for Best Practices (NGA Center) and the Council of Chief State School Officers (CCSSO), 2010) found at: <http://www.corestandards.org/>

Below is just a sampling of elementary level objectives from the new Common Core State Standards (CCSS) that can be addressed using the GeoGebra software.

See Appendix B for examples on using GeoGebra and photography to create meaning and understanding of geometry for the students.

Creating Confident Math Teachers to use Geogebra with their Students

GeoGebra was described as raising the enthusiasm for the effective and wise application of technology to the teaching/learning enterprise (Fahlberg-Stojanovska and Stojanovski, 2009; Hewson, 2009). Observations of participants in schools and during the summer workshops are also cited as evidence. GeoGebra was also credited with changing teacher habits. Two features were specifically referenced as causing this change: 1) that it is an award winning software system, and therefore has admirable features, and 2) that it provides an effective pedagogical model for teachers.

The aspects of GeoGebra that make it award winning, insofar as those aspects are related to changing teacher habits, were described as supporting teacher demonstration, allowing students real-time exploration opportunities, and that even if the GeoGebra software is used and eliminated from instruction that it will have raised teachers' expectations and standards for future technology-use. They also noted that these are benefits, at least in part, unique to GeoGebra.

As described by Mishra and Koehler (2006) Technological Pedagogical Content Knowledge (TPCK) is the basis of good teaching with technology and requires not only content knowledge or pedagogical knowledge but an understanding of the representation of concepts using technologies, how to teach these math concepts using technology, knowledge on the challenges their students will face when presented with this new pedagogy, and how technology can be used to build on existing knowledge and develop new knowledge. Scandrett (2008) feels that math teachers need to always start by using concrete models in geometry using manipulatives like geoboards which provide a concrete model of understanding. Rosen & Hoffman (2009) have found that teachers need to connect students understanding from the concrete to abstract and using virtual manipulatives and software like GeoGebra better help make those connections to representational models connecting the concrete using geoboards to something even more abstract in understanding.

With the availability of dynamic mathematics software, like GeoGebra, teachers are able to make graphical representations of math concepts. As the concepts are introduced with pictorial representations, teachers and their students are able to make the connections between the pictures, the math concepts, and the symbolic representation. When presented with a new concept, students need to think, visualize and explore relationships and patterns. This is consistent with the CRA (Concrete, Representational, and Abstract) Model for teaching mathematics currently in better



reaching students as they learn and understand mathematical concepts. Technology makes all of this possible for them in a short amount of time.

Math teachers need to ask themselves some tough questions when it comes to professional development and employing technology into their teaching of mathematics, things like:

1. What role does technology play in providing multiple representations and opportunities for communication to help students develop mathematical understanding?
2. How does technology influence your instructional decisions? And, how do your instructional decisions influence your use of technology?
3. How can technology increase access to significant mathematics to all students? How do you promote social justice for access to and facility with technology in learning mathematics?
4. How are you thinking differently about your use of technology as a result of reading this article on GeoGebra? What are some of the steps you plan to take to promote growth in your own use of technology?

Professor Freedman Provides Math Help at: <http://www.mathpower.com/>

- Math Teachers' Ten Commandments
- Math Anxiety Self-Test
- Ten Ways to Reduce Math Anxiety
- Students' Math Bill of Rights
- Study Skills Tips
- Math Anxiety Code of Responsibilities
- Other Links to Math Help

At the Mathitudes Online website: <http://www.coe.fau.edu/mathitudes/>, one can find a multitude of web links related to math anxiety research. A famous quote from W. V. Williams (1988), "Tell me mathematics, and I will forget; show me mathematics and I may remember; involve me...and I will understand mathematics. If I understand mathematics, I will be less likely to have math anxiety. And if I become a teacher of mathematics, I can thus begin a cycle that will produce less math-anxious students for generations to come." Is a reminder of how critical it is to teach for understanding making things as hands-on and real-world as possible? "If math teachers do something about helping their students to develop their confidence and ability to do math, we can impact their lives in a positive way forever" and "Our students' careers and ultimately many of their decisions they will make in life could rest upon how we decide to teach math. We must make the difference for the future of our kids in an ever growing, high-tech, competitive, global world which depends so heavily on mathematics." (Furner, 1999)

Summary

So why should math teachers use GeoGebra? The first reason is because it is free to download and use, one can go to GeoGebra.org. GeoGebra is an up and coming dynamic teaching tool in our schools today. It is user-friendly for students and teachers. It is great way to connect from the hands-on Geoboards to virtual Geoboards to something even more abstract. GeoGebra.org provides many resources and teaching tools at its wiki for educators at: http://www.GeoGebra.org/en/wiki/index.php/Main_Page. One of the best reasons for using GeoGebra is that it can even be used for primary-aged students through college, it is fun, easy to use, and students learn a lot about geometry, algebra, measurement and beyond by using this dynamic tool.

We can summarize this idea of photography and GeoGebra by saying:

- To show a purpose for math
- To develop relationships
- To show practical applications to math in life
- To make connections
- To employ innovative teaching
- To stimulate through photography/Modeling
- To employ emerging technologies in math with the real world

Some PowerPoints and Data Files for GeoGebra as they relate to this paper can be accessed at: <http://matharoundus.com/>, there are many resources here for math teachers Grades K-12 to download for free. We hope you will consider using this dynamic tool in your classroom to teach math and turn more students onto the STEM fields. Young learners intrigued by technology will construct and investigate geometric shapes with GeoGebra and start liking and enjoying math more!

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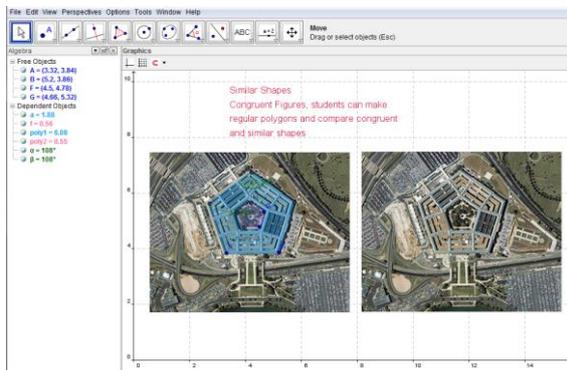
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Appendix A: Geogebra and Virtual Manipulative Websites and Resources for the Classroom

National Library of Virtual Manipulatives	http://nlvm.usu.edu/en/nav/topic_t_3.html
National Council of Teachers of Mathematics	http://www.nctm.org/standards/content.aspx?id=25007
Cut the Knot	http://www.nctm.org/standards/content.aspx?id=25007
Geoboard Resources	http://msteacher.org/epubs/math/QuickTakes/geoBoard.aspx
GeoGebra	http://GeoGebra.org
GeoGebra Wiki Forum	http://www.GeoGebra.org/en/wiki/index.php/Main_Page
GeoGebra Data Files	http://matharoundus.com/GeoGebra
Math Academy	http://www.mathacademy.com/pr/mini/text/anxiety/
Mathitudes Online	http://www.coe.fau.edu/mathitudes/

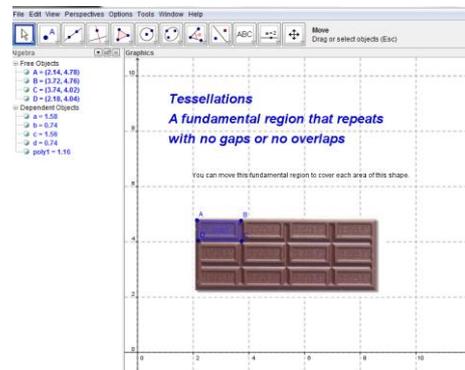
Appendix B: K-6 Math Topics

Similar Shapes



Similar Shapes
Congruent Figures, students can make regular polygons and compare congruent and similar shapes.

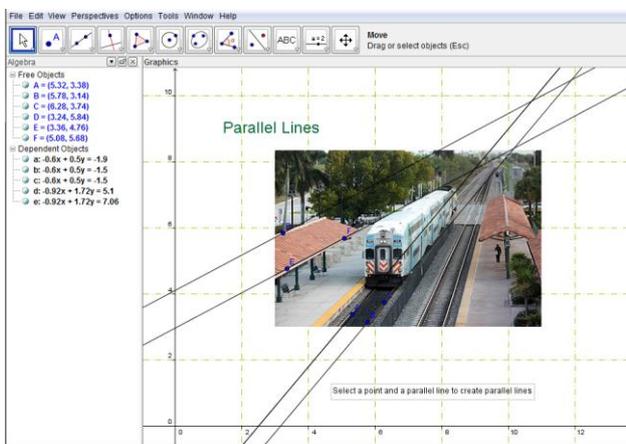
Tessellations



Tessellations
A fundamental region that repeats with no gaps or no overlaps

You can move this fundamental region to cover each area of this shape.

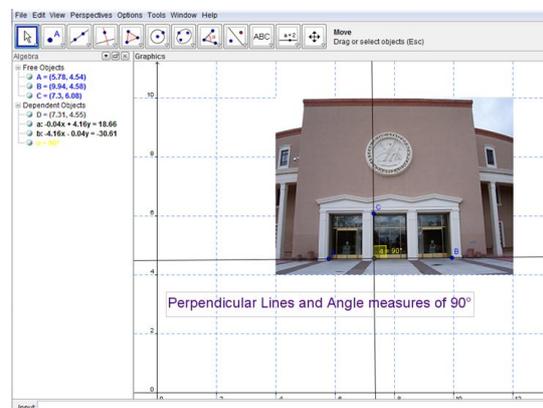
Parallel Lines



Parallel Lines

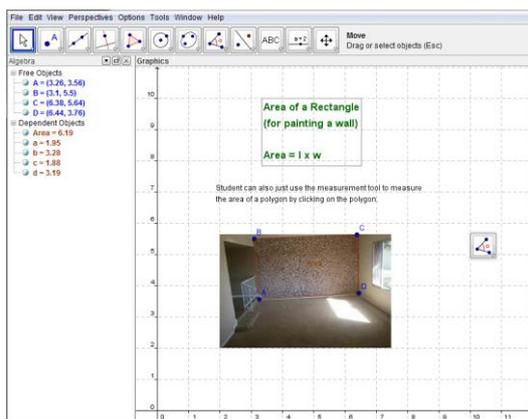
Select a point and a parallel line to create parallel lines

Perpendicular Lines



Perpendicular Lines and Angle measures of 90°

Area

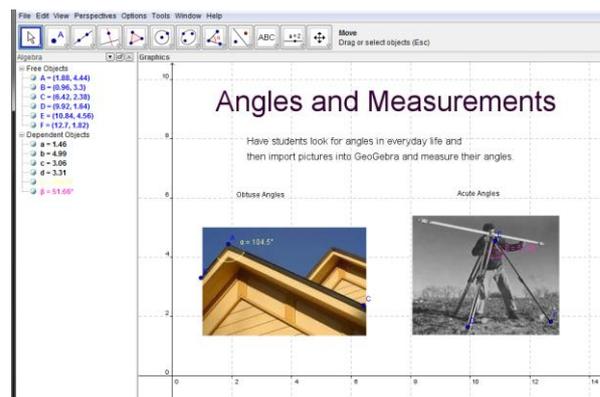


Area of a Rectangle (for painting a wall)

Area = $l \times w$

Student can also just use the measurement tool to measure the area of a polygon by clicking on the polygon.

Angles and Measures



Angles and Measurements

Have students look for angles in everyday life and then import pictures into GeoGebra and measure their angles.

Obtuse Angles
Acute Angles



About the Authors:



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Joseph M. Furner, Ph.D., is Professor of Mathematics Education in the Department of Teaching and Learning in the College of Education at Florida Atlantic University in Jupiter, Florida. His research interests are related to math anxiety, the implementation of the NCTM and the Common Core State Mathematics Standards, TESOL issues as they relate to math instruction, the use of technology in mathematics instruction, the use of math manipulatives, and children's literature in the teaching of mathematics. Dr. Furner is the author of 60+ publications and the author of the book, *Living Well: Caring Enough to Do What's Right*. He has worked as an educator in New York, Florida, Mexico, and Colombia.



Ana Escuder, Ph.D., has had a long tenure of experience in mathematics instruction at many levels including 10 years in a middle school, 8 years in a high school, and 9 years in higher education. She holds a Ph.D. degree in curriculum and instruction with a specialization in mathematics from Florida Atlantic University (FAU) where she currently works with undergraduate students and in-service mathematics teachers while pursuing interests in the facilitation of mathematics learning and comprehension through the use of interactive technology. She has had the fortune of working to pursue and manage several federally funded grants at the university in mathematics education and curriculum development for middle school and high school teachers, and is currently the director of the GeoGebra Institute at FAU.