TEACHING MATH CONCEPTS THROUGH HISTORICAL LOCATIONS USING GEOGEBRA AND PHOTOGRAPHY

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Abstract

By exploring objectives related to the new Common Core Math Standards through using photographs with GeoGebra software, students can learn about mathematics and even social studies and geography concepts together. The topics will explore the math that surrounds us thus creating a connection between the abstract math and the real-world experiences. Historical photographs that are inserted into GeoGebra will provide the basis to observe relationships with different and similar shapes from different geographic locations. Interactive technology like GeoGebra can help motivate learners to enjoy learning mathematics while connecting both mathematics and social studies concepts. The paper will show educators how by importing photography into the GeoGebra software, teachers can explain math concepts and make the teaching and learning of mathematics more real-world relevant while students also can learn geography and history at the same time. In an age of STEM, it is critical that we motivate and turn young people on to math through technology and real-world connections.

Key Words: Teaching Mathematics, GeoGebra, Photography, Geography, History, Common Core Mathematics Standards, Real-world

Introduction

When using photography along with GeoGebra, we can better connect and show our students how math surrounds us. In today's technologically-oriented world, students need to be skilled in Science, Technology, Engineering, and Mathematics (STEM) fields. By the National Council of Teachers of Mathematics (NCTM, 2000) endorsement and the new Common Core State Standards (CCSS) in Mathematics emphasis, it is necessary that we teach using technology, encourage connections to other disciplines like social studies, and make the math that students are learning relevant and meaningful. Geometry is the main branch of mathematics that relates to the visual aspects of our surroundings. By relating geometric shapes and relationships to historical photographs, students not only recognize the shapes but understand their purpose and how they are created. This paper looks at ideas for teaching mathematics with the use of technology, photography,

and content integration to social studies using the free dynamic mathematics software, GeoGebra, to provide connections and motivation in learning mathematics.

2015 Pi Day Example Lesson

It is exciting to teach math today using technology and images like photographs. We started our presentation for the 2015 Pi Day sharing a photo of a very large pie, the largest pumpkin pie in the world. According to the Guinness World Records (2010), "*The largest pumpkin pie weighs 1,678 kg (3,699 lb) and was made by New Bremen Giant Pumpkin Growers (USA) at New Bremen Pumpkinfest in New Bremen, Ohio, USA, on 25 September 2010. The diameter of the pie was 6 m (20 ft). The crust was made of 440 sheets of dough and the other ingredients were canned pumpkin, evaporated milk, eggs, sugar, salt, cinnamon and pumpkin spice."* Information found at: http://www.guinnessworldrecords.com/world-records/largest-pie-pumpkin

Figure 1: Largest Pumpkin Pie

Educators need to incorporate motivational activities like Pi Day into their classroom activities. Math teachers can implore the emerging technologies like GeoGebra in a math classroom and import photos to powerfully provoke thinking and learning.

Historical Locations and Mathematical Shapes

The presentation continued with the participants seeing the below questions and photos:

- Can you identify where each one of these photos was taken?
- Can you give a historical account that took place for each photo?
- Can you see some mathematical idea within the photo?

Figure 2: Three Historic Locations with Geometer Shapes

In research from Zeynep and Nesrin (2010), the use of geometry software supported by digital daily life photographs on geometry learning had positive effects on mathematics achievement. Their findings demonstrate that using dynamic software like GeoGebra supported by digital photographs results in improvements in achievement, permanence of knowledge, and relating mathematics subjects to daily life. This is the theoretical framework/premise of this paper and research.

When the participants knew the geographic locations based on the photos, it motivated them to connect historical locations and explore what math concepts are in each photo. Mathematics teachers today can use such images and photos in their teaching to stimulate active learning while also making connections to other disciplines like social studies and geography. Math teachers can use photos and images to stimulate the learning of many math concepts while also covering social studies concepts.

Using Photography to Teach Mathematics

Munakata and Vaidya (2012) based on their research found that students do not consider mathematics and science to be creative endeavors, though the traditional artistic disciplines rank high in this regard. To address this problem in perception, the authors used photography as a means to encourage students to find the deep-rooted connections between science, mathematics and the arts. The photography project was used in a formal classroom setting as well as an outside activity, i.e. in a more informal setting. The project found student interest and motivation were peaked when photography was part of the instructional strategies to teach new material while making meaningful connections to the math using the photography. Math teachers should encourage students to take photos that include math concepts (Northcote, 2011). This idea works well with the authors' suggestions of having students import photos into GeoGebra and then use the software to point out important mathematical ideas.

The Social Studies and Mathematics Connection

Kinniburgh & Byrd (2008) have found and advocated connecting the teaching of mathematics and social studies, they suggested that using such connections makes deeper understanding for students as they learn. Today we can even use images and photographs to make connections where students can see both the math and social studies concepts within the photos. Newcombe (2013) says, "Science, mathematics, and social studies are deeply spatial subjects. Currently, students who come to class with higher levels of spatial ability take more easily to learning in these areas, but this fact does not mean we cannot teach in a way that maximizes learning for all. Spatial ability can be improved both inside and outside the classroom, as well as by instruction in other subject areas, notably the visual arts. Spatializing the curriculum by including and explicitly teaching the spatial symbol systems that lie at the heart of science, mathematics, and social studies is an achievable and worthwhile goal (p.31)."

Relating and understanding real-world problems through the use of interactive technology like GeoGebra and connecting them to photography will create important connections in math and social studies by making learning more meaningful for learners.

GeoGebra

GeoGebra is a multi-platform dynamic mathematics software for all levels of education from elementary through university that joins dynamically geometry, algebra, tables, graphing, spreadsheets, statistics and calculus in one easy-to-use package (Hewson, 2009; Hohenwarter, Hohenwarter, & Lavicza, 2009). This open-source dynamic mathematics software can be downloaded for free and materials accessed at: http://www.geogebra.org/cms/en/info. There are no licensing issues associated with its use, allowing students and teachers freedom to use it both within the classroom and at home. GeoGebra has a large international user and developer community with users from 190 countries is currently translated into 55 languages.

Fahlberg-Stojanovska, & Stojanovski (2009) found that using GeoGebra for motivating students and through exploration and conjectures learn at a higher level. Although GeoGebra has been primarily intended for mathematics instruction in secondary schools, it certainly has uses in higher education and even now introduced in the elementary math levels. The Appendix A provides online websites on resources related to GeoGebra.

GeoGebra can show how mathematical equations can be applied to everyday objects. Aydin & Monaghan (2011) in their research feel that math teachers need to explore the potential for students to "see" mathematics in the real world through "marking" mathematical features of digital images using a dynamic geometry system like GeoGebra. Mathematics teachers may find the following videos (Mathematics and Multimedia, n.d.) of basic training for GeoGebra at: http://mathandmultimedia.com/2011/01/01/geogebra-essentials-series/ useful as they provide great resources for how to quickly use GeoGebra in their classrooms.

Research from Zengin, Furkan, & Kutluca (2012) has found that using GeoGebra to teach math has been proven to increase achievement scores in mathematics. The results of the study indicated that there was a significant difference between the means of the students' scores on the posttest in favor of the GeoGebra group when teaching mathematical concepts. Tatar (2012) also found a positive impact of utilizing dynamic mathematical learning software like GeoGebra thus enhancing students learning and understanding. More information and the free download can be found at: http://www.geogebra.org/cms/en/info

What Math do you see in these Photos?

It is important we encourage our learners to recognize that geometry and shapes/mathematics surround us! Aydin & Monaghan (2011) and Bragg & Nicol (2011) in their research show that teaching math with photography and technology like

GeoGebra is motivational for students. It helps to make personal real-life interactions that connects learners with mathematics in important ways.

The quote by Walt Disney (n.d.), *"Of all of our inventions for mass communication, pictures still speak the most universally understood language,"* says it all. This paper shares historical sites and images that could be used to cover a large number of mathematical concepts. Jones (2012) in her book, *Visualizing Mathematics*, discusses how teachers need to help students visualize and create representations of their math understanding to electrify them on the subject. Below and in Appendix B include many example of such math concepts that can be taught within photos.

Examples of Mathematical Concepts found at Historical Sites:

- Circles
- Parabolas
- Parallel Lines
- Perpendicular Lines
- Pythagorean Theorem
- Reflections
- Similar Shapes
- Spirals
- Symmetry
- Tessellations

Covering the Common Core Math Standards/State Standards while Connecting Math and Social Studies

As set forth by the new Common Core Math Standards (National Governors Association Center for Best Practices (NGA Center) and the Council of Chief State School Officers 2010 (CCSSO), most schools and states are adhering the guidelines. (http://www.corestandards.org/)

See Appendix B for examples on using GeoGebra and photography to connect and understand geometry. Below are examples of Common Core Math Standards that can be covered with the photos and GeoGebra examples used with this paper:

CCSS.MATH.CONTENT.K.G.A.2
Correctly name shapes regardless of their orientations or overall size.

CCSS.MATH.CONTENT.2.G.A.1
Recognize and draw shapes having specified attributes, such as a given number of angles or a given number of equal faces.1 Identify triangles, quadrilaterals, pentagons, hexagons, and cubes.

CCSS.MATH.CONTENT.4.G.A.1
Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures.

CCSS.MATH.CONTENT.6.G.A.3
Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates

to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems.

CCSS.MATH.CONTENT.8.G.B.7

Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.

CCSS.MATH.CONTENT.HSG.CO.A.1

Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.

By relating and understanding real-world problems through the use of dynamic technology and connecting them to photography to make important connections in math, our learners recognize that geometry and shapes/mathematics surround us! The above Common Core Math Standards can all be taught using GeoGebra and photographs. See Appendix B and the website, www.matharoundus.com for more ideas.

Integrating Mathematics, Social Studies, Photography, and Technology

Edutopia (2008), great advocate of integrated studies for teaching mathematics, says *"Integrated studies, sometimes called interdisciplinary studies, brings together diverse disciplines in a comprehensive manner, enabling students to develop a meaningful understanding of the complex associations and influences within a topic. A happy by-product of this approach, which is often coupled with project-based learning, is that it makes school more interesting and productive for students and teachers* (Para. 2)."

Curriculum integration can help teachers cover the required social studies and mathematics standards that students will be taught, in the same lessons (Sahinkaya & Aladag, 2010). Researchers feel that mathematics is a subject that teachers seem to have the most difficulty integrating with other content areas but the best connection is with social studies.

The use of technological tools is critical in today's world. Our students need to learn to excel at higher levels of generalization, model and solve complex problems, and focus on decision-making and reasoning (National Council of Teachers of Mathematics (NCTM) 1989, 2000, 2006). NCTM believes that mathematical power can arise from technology that includes: increased opportunity for learning, increased opportunities for real-life social contexts, and orientation to the future. The President's Council of Advisors on Science and Technology (PCAST) (Holdren, Lander, & Varmus, 2010) released an executive report in November 2010 where specific recommendations to the administration are given to ensure that the United States is a leader in Science, Technology, Engineering, and Mathematics (STEM) education in the coming decades.

One recommendation is to recruit and train 100,000 new STEM middle and high school teachers over the next decade that are able to prepare and inspire students to have strong majors in STEM fields and strong content-specific pedagogical preparation. PCAST regards teachers as the most important factor in ensuring excellence in STEM education.

History, Mathematics, and Photography: Albert Einstein (Einstein, n.d.)

Quotes by Albert Einstein related to history, mathematics, and photography:

Math

"Do not worry about your difficulties in Mathematics. I can assure you mine are still greater."

Geography/History

"In the teaching of geography and history a sympathetic understanding (should) be fostered for the characteristics of the different peoples of the world, especially for those who we are in the habit of describing as "primitive."

Photography

"A photograph never grows old. You and I change, people change all through the months and years but a photograph always remains the same. How nice to look at a photograph of mother or father taken many years ago. You see them as you remember them. But as people live on, they change completely. That is why I think a photograph can be kind."

Einstein was well-regarded in the science world. His quotes are meaningful and everlasting. His thoughts on math understanding and he also saw value in so much more in life. The above quotes really touch home with educators in making such connections. Today educators need to seek out innovative ways to teach mathematics to excite and motivate students using technology and photography to teach mathematics in meaningful ways making such connections (Furner & Marinas, 2014). Educators can glean a great deal from Einstein's wisdom and in how we should go about teaching math so to excite learners and maybe perhaps maybe attract more young people in the STEM fields.

Summary

Teaching math using GeoGebra, photography, and making connections to social studies and mathematics can:

- Show a purpose for math
- Make connections and relationships between math and social studies by teaching some history while covering math concepts
- Employ emerging technologies in math with the real world
- Show practical applications to math in life
- Employ innovative teaching in the classroom
- Stimulate excitement through Photography/Modeling

Math learners fascinated by technology will construct and investigate geometric shapes and many math ideas with GeoGebra and will start enjoying math and even see connections to history/geography/social studies! By using technology like GeoGebra and photographs, our learners are intrigued by technology through their constructions and investigations of geometric shapes with GeoGebra and start liking and enjoying mathematics more!

PowerPoint presentations and Data Files for GeoGebra can be accessed at: http://matharoundus.com/. Many free resources for math teachers Grades K-12 to download are in Appendix A.

References

Aydin, H., & Monaghan, J. (2011). Bridging the divide--Seeing mathematics in the world through dynamic geometry. *Teaching Mathematics and Its Applications: An International Journal of the IMA*, 30(1), 1-9.

Bragg, L. A. & Nicol, C. (2011). Seeing mathematics through a new lens: Using photos in the mathematics classroom. *Australian Mathematics Teacher,* 67(3), 3-9. Retrieved from http://dro.deakin.edu.au/eserv/DU:30036898/bragg-seeingmathematics-2011.pdf

Disney, Walt. (n.d.) Walt Disney Quotes. Retrieved on April 10, 2015 at: http://www.justdisney.com/walt_disney/quotes/

Edutopia, (2008). Why should schools embrace integrated studies?: It fosters a way of learning that mimics real life. Retrieved April 10, 2015 at: http://www.edutopia.org/integrated-studies-introduction

Einstein, A. (n.d.) Kata-kata bijak Albert Einstein – My Blog. Retrieved April 10, 2015 at: http://hermawayne.blogspot.com/2009/04/kata-kata-bijak-albert-einstein.html

Fahlberg-Stojanovska, L, & Stojanovski, V. (2009). GeoGebra- freedom to explore and learn. *Teaching Mathematics and Its Applications: An International Journal of the IMA*, 28(2), 49-54.

Furner, J. M., & Marinas, C. A. (2014). Addressing math anxiety in teaching mathematics using photography and GeoGebra. Paper presented at *The International Conference on Technology in Collegiate Mathematics Twenty-sixth Annual Conference*, San Antonio, Texas, March 22, 2014 (pp. 134-143).

GeoGebra Free Software Download, (n.d.) Retrieved on April 27, 2015 at: http://www.geogebra.org/cms/en/

Guinness World Records. (2010).World's largest pumpkin pie. Retrieved on March 13, 2015 at: http://www.guinnessworldrecords.com/world-records/largest-pie-pumpkin

Hewson, P. (2009). GeoGebra for mathematical statistics. *International Journal for Technology in Mathematics Education*, 16(4), Retrieved May 5, 2011 at: http://www.editlib.org/p/30304

Hohenwarter, J., Hohenwarter, M., and Lavicza, Z. (2009). Introducing dynamic mathematics software to secondary school teachers: The case of GeoGebra. *The Journal of Computers in Mathematics and Science Teaching*, 28(2), 135-46.

Holdren, J., Lander, E., & Varmus, H. (2010). Prepare and inspire: K-12 education in science, technology, engineering and math education for America's future. The President's Council of Advisors on Science and Technology, Office of Science and Technology Policy. Retrieved May 5, 2011 at: http://www.whitehouse.gov/administration/eop/ostp/pcast/docsreports

Jones, J. C. (2012*). Visualizing: Elementary and middle school mathematics methods.* Hoboken, NJ: John Wiley and Sons, Inc.

Kinniburgh, L. H. & Byrd, K. (2008). Ten black dots and September 11: Integrating social studies and mathematics through children's literature. *The Social Studies*, 99(1), 33-36. doi: 10.3200/TSSS.99.1.33-36

Mathematics and Multimedia. (n.d.) Mathematics and multimedia K-12 mathematics teaching and learning through multimedia: GeoGebra essentials series. Retrieved on April 10, 2015 at: http://mathandmultimedia.com/2011/01/01/geogebra-essentials-series/

Munakata, M., and Vaidya, A. (2012). Encouraging creativity in mathematics and science through photography. Teaching Mathematics and Its Applications: An International Journal of the IMA, 31(3), 121-132.

National Council of Teachers of Mathematics. (2006). Curriculum focal points for prekindergarten through grade 8 mathematics: a quest for coherence. Reston, VA.:Author.

National Council of Teachers of Mathematics. (2000). Principles and Standards for School Mathematics. Reston, VA: Author. National Council of Teachers of Mathematics. (1989). NCTM Curriculum & Evaluation Standards. Reston, VA: NCTM.

National Council of Teachers of Mathematics. (1989). Curriculum and evaluation standards for school mathematics. Reston, VA: Author.

National Governors Association Center for Best Practices (NGA Center) and the Council of Chief State School Officers (CCSSO) (2010). *Common core state standards initiative*. Washington, DC. Authors. The Common Core State Standards may be accessed and/or retrieved on November 14, 2010 from http://www.corestandards.org.

Newcombe, N. S. (2013). Relationships using spatial thinking to teach science, mathematics, and social studies, *American Educator,* 37(*1),* 26-31, 40.

Northcote, M. (2011). Teaching with technology: Step back and hand over the cameras! Using digital cameras to facilitate mathematics learning with young children in K-2 classrooms. *Australian Primary Mathematics Classroom, 16*(3), 29-32. Retrieved from http://research.avondale.edu.au/cgi/viewcontent.cgi?article=1024&context=edu_papers

Sahinkaya, N. & Aladag, E. (2010). The views of class teachers on the integration between social studies and mathematics. *Procedia: Social and Behavioral Sciences*, 2, 2876-2880. doi:10.1016/j.sbspro.2010.03.432

Tatar, E., (2012). The effect of dynamic mathematics software on achievement in mathematics: The case of trigonometry. *Energy Education Science and Technology Part B: Social and Educational Studies.* 4 (1), 459-468.

Zengin, Y., Furkan, H., & Kutluca, T. (2012). The effect of dynamic mathematics software geogebra on student achievement in teaching of trigonometry. *Procedia: Social and Behavioral Sciences,* 31, 183–187. doi:10.1016/j.sbspro.2011.12.038

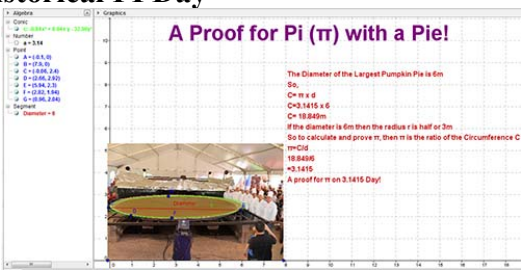
Zeynep, G. & Nesrin, O. (2010). The effects of using geometry software supported by digital daily life photographs on geometry learning. *Procedia: Social and Behavioral Sciences*, 2(2), 2824–2828. doi:10.1016/j.sbspro.2010.03.422

Appendix A: GeoGebra Websites and Resources for the Classroom

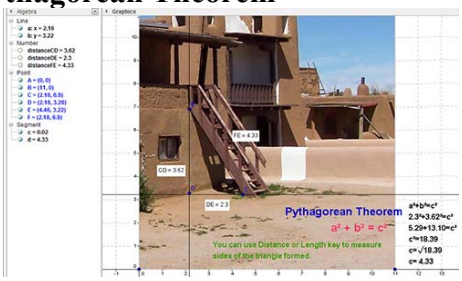
Geoboard Resources	http://msteacher.org/epubs/math/QuickTakes/geoBoard.aspx
GeoGebra	http://GeoGebra.org
GeoGebra Wiki Forum	http://www.GeoGebra.org/en/wiki/index.php/Main_Page
GeoGebra Data Files	http://matharoundus.com
Math Academy	http://www.mathacademy.com/pr/minitext/anxiety/
Mathitudes Online	http://www.coe.fau.edu/mathitudes/

Appendix B: Sample Math and Geography/Historical Location GeoGebra Files

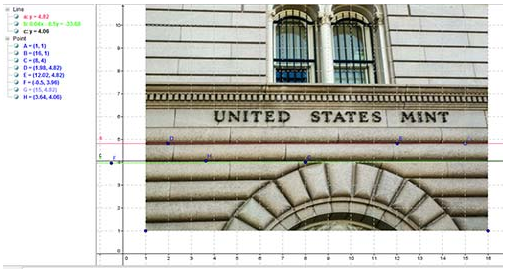
Historical PI Day



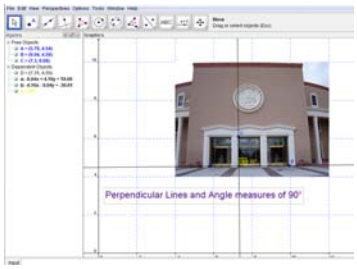
Pythagorean Theorem



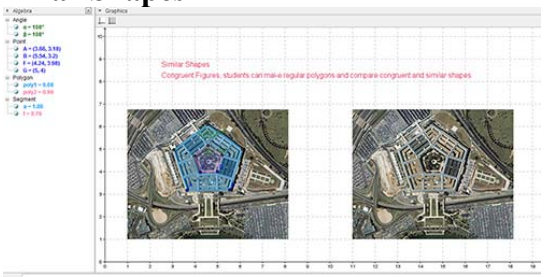
Parallel Lines



Perpendicular Lines



Similar Shapes



Tessellations