CONNECTING GEOMETRY, MEASUREMENT, AND ALGEBRA USING GEOGEBRA FOR THE ELEMENTARY GRADES

Joseph M. Furner, Ph.D.
Florida Atlantic University
5353 Parkside Drive, Ed 207D
Jupiter, Florida 33458
E-mail: jfurner@fau.edu

and

Carol A. Marinas, Ph.D.
Barry University
11300 NE 2nd Avenue
Miami, Florida 33161
E-mail: drmarinas@yahoo.com

Abstract

This paper shares many examples of connections to geometry, measurement, and algebra using GeoGebra. Through the mathematical and pedagogical power of GeoGebra, the learner will use this user-friendly software to make meaningful connections for K-6 students by exploring and creating geometry figures and ideas and then continuing to measure such figures with the GeoGebra software tools. Using the algebra view and tools from the software, students will see the formulas or algebraic representations for generalizing a wide variety of geometric figures and their measurements in an abstract algebraic representation. Since this software is available as a free download, many school districts are adopting this software in their classrooms as it is a powerful tool for teaching and connecting geometry, measurement, and algebra.

Key Words: Teaching Mathematics, GeoGebra, Geometry, Measurement, Algebra, Technology, Standards

Introduction

In a globally competitive world, it has never been more important than now to better prepare our students for science, technology, engineering, and mathematics (STEM) fields. While using the software, GeoGebra, math teachers can cover all areas of STEM as it applies to math, science, and engineering ideas. The National Council of Teachers of Mathematics (NCTM) has made technology a main focus for teaching math in today's classrooms. GeoGebra can help to create connections from geometry, to measurement, and algebra. Through the mathematical and pedagogical power of GeoGebra, the learner will use this user-friendly software to make meaningful connections for K-6 students by exploring and creating geometry figures and ideas and then going on to measure mathematical relationships with the GeoGebra software tools. Using the algebra tools

from the software, students will see the formulas or algebraic representations for generalizing a wide variety of geometric figures and their measurements in an abstract algebraic way. This paper shares ideas for teaching K-6 elementary students GeoGebra activities for connecting geometry, measurement, and algebra to each other.

Using Technology to Motivate Students

Research has shown over the years from many famous researchers in math education that it is important to have students involved interactively in the classroom (NCTM, 1989, 2000, 2006). Jean Piaget, Marie Montesorri, and Marilyn Burns have all been true leaders in overhauling the teaching of mathematics by insisting on teaching using hands-on manipulatives and making connections from the hands-on, to the representational, to learners understanding math at an abstract level. Today most educators support mathematics classroom where students are involved and actively constructing their own understanding of mathematical ideas while using the latest emerging technologies.

Today the emphasis is on using technology to teach math and getting more students interested in science, technology, engineering, and mathematics (STEM) fields. Using software like GeoGebra is a great way to teach all the important STEM-related topics while also motivating students when learning and covering the new math standards we are covering in our schools.

According to the NCTM Standards (2000) the use of technological tools, students can:

- work at higher levels of generalization
- model and solve complex problems
- focus on decision-making and reasoning

Mathematical power can arise from technology that includes:

- increased opportunity for learning
- increased opportunities for real-life social contexts
- orientation to the future

GeoGebra

While the study of geometry in some shape or form has existed for many millennia, it is within the past twenty years that a shift has occurred in how geometry may be learned through computer-based interactive geometric software. Software programs like GeoGebra allow users to construct interactive representations of points, lines, and circles. These geometric objects are interactive in that they may be resized and shifted around onscreen through clicking and dragging actions. Moreover, interactive geometric software like GeoGebra in the K-12 mathematics curriculum has been used at the elementary level, middle school level, and the high school level (Yu & Tawfeeq, 2011) and has proven to be a very effective means for teaching and learning mathematics.

Fahlberg-Stojanovska, & Stojanovski (2009) found that using GeoGebra motivates and helps students learn at a higher level while exploring and conjecturing as they draw and measure. Rosen & Hoffman (2009) found that it is very important to integrate both concrete and virtual manipulatives into the primary-age math classroom. Furner & Marinas (2007) found that children easily transition to the abstract when using geometry sketching software when you first use geoboards and the software like *Paint* before going directly to the sketching software. Appendix C shows several examples young children can explore with while using GeoGebra.

GeoGebra is a great resource and technological tool that when used in the math classroom provides a focus to:

- promote technology as an essential tool for learning mathematics in the 21st century
- integrate the principles and process standards with teaching the content standards
- provide access to all five mathematics content standards for all students
- support learner-centered strategies that address the diverse needs of all learners of mathematics

GeoGebra allows students to actively constructing their own understanding of geometry, measurement, and algebra using this technology. With GeoGebra to meet the Common Core Math Standards, students can do many things such as:

- use the polygon and circle tools to draw shapes
- measure angles and distance
- use GeoGebra sliders to adjust values of different problems
- insert images into a file to demonstrate mathematical problem solving
- recognize perimeter as an attribute of plane figures and distinguish between linear and area measures
- reason with shapes and their attributes

Effective mathematics teachers can maximize the potential of GeoGebra to:

- develop students' understanding
- stimulate their interest
- increase their proficiency in mathematics

GeoGebra is free software, a multi-platform dynamic mathematics software for all levels of education that joins geometry, algebra, tables, graphing, statistics and calculus in one easy-to-use package (Hohenwarter, Hohenwarter, & Lavicza, 2009). GeoGebra has a large international user and developer community with users from 190 countries. The software is currently translated into 55 languages and attracts close to 300,000 downloads per month. It can be downloaded for free and accessed at: http://www.geogebra.org/cms/en/info.

Connections in Teaching Mathematics

How can a teacher use the practice of reflection to create rich mathematical learning environments that are engaging to students? In such environments, one can hear and see a seamless integration of Problem Solving, Reasoning and Proof, Communication, making Mathematical Connections, and Representation (NCTM Process Standards, 2000) through Number and Operations, Algebra, Geometry, Measurement, and Data Analysis and Probability. While the five Content Standards are the heart of the curriculum, the Process Standards are essential to keeping it flowing. If the Mathematics Content Standards are "what" teachers teach, the Process Standards are "how" teachers teach it and how students learn it. Although the Process Standards are a required part of the state curriculum, many teachers need ways to make sense of them within the context of their teaching.

Connections between the Common Core State Standards and the National Council of Teachers of Mathematics (NCTM) Standards need to meet the challenges of differentiating mathematics instruction in the K-8 classroom. Small (2012) explains two powerful and universal strategies that teachers can use across all math content: Open Questions and Parallel Tasks. Showing teachers how to get started and become experts with these connection strategies, Small also demonstrates more inclusive learning conversations that promote broader student participation and mathematical thinking. Guidance for creating a more inclusive classroom learning community with mathematical talk that engages participants from all levels is important (Small, 2012). While using the technology and covering all the required math standards by making all necessary connections for sound learning, students can work together and use the software to engage in higher level mathematical understanding.

Andresen & Misfeldt (2010) found that with the new Common Core Standards (National Governors Association Center for Best Practices - NGA Center, 2010) in teaching mathematics, teachers need to be trained and learn new mathematics content and technology like GeoGebra in order to be effective in teaching and reaching their students. Knowledge of technology cannot be isolated from the content and good mathematics teaching requires an understanding on how technology is related to the pedagogy and mathematics (Hughes, 2005).

Today, most schools and states are adhering to the new Common Core Math Standards (National Governors Association Center for Best Practices (NGA Center) and the Council of Chief State School Officers (CCSSO), 2010) found at: http://www.corestandards.org/. A sampling of objectives from the new Common Core Standards that can be addressed using the GeoGebra software follows.

A Look at the New Common Core Standards

Common Core Math Standards http://www.corestandards.org/

Grade 3

3.G.1. Understand that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories.

3.G.2. Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole. For example, partition a shape into 4 parts with equal area, and describe the area of each part as 1/4 of the area of the shape.

Grades 6

6.G.1. Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.

6.G.3. Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems.

Levels of Understanding New Concepts

1. Start with the Concrete using hands-on manipulatives, like Geoboards
2. Move to Representational models in diagrams, like pictures of Virtual Manipulatives
3. Lastly, connect to the Abstract symbolism where student understand and function at an abstract level completely connecting geometry, measurement, and algebra when using GeoGebra.

NCTM has advocated problem solving as the central theme of the math curriculum for many years now (NCTM, 1989, 2000, 2006). In order to accomplish the Common Core Standards, the authors believe that students should use GeoGebra to simulate and solve word problems and explore conjectures that lead to formulas and theorems. By working in groups or alone, students explore weekly problems by using the GeoGebra software to show their true understanding of the math concepts in a representational manner. With GeoGebra covering many of the new Common Core Math Standards, it lends itself well to learn and connect many geometry, measurement, and algebra standards and concepts.

An Example Math Problem using Geometry, Measurement, & Algebra

Example Word Problem which can be modeled in GeoGebra:
On Japanese trains, there is a rule that forbids passengers bringing onto the train objects longer than 36 inches. How did a passenger travel with a ceremonial sword that was 42 inches long? Discuss strategies and solutions (See Appendix C).

Today it is critical that as teachers, we connect the geometry and measurement integrated with algebra to better prepare them for higher level mathematics. Connections and technology are both critical aspects for learning mathematics and part of NCTM's principles for teaching mathematics (2000, 2006). Everything that is geometric we want to measure. All measurements can by synthesized into formulas that use algebra and algebraic expressions. While motivating students in learning mathematics, educators must make these connections. When using software like GeoGebra, it is much easier making the connections. The Appendices offer many other suggested GeoGebra activities and other mathematical connections/resources that can be used by classroom math teachers.

Summary

So why should elementary mathematics teachers use GeoGebra in their classrooms to teach mathematics? The first reason is because it is free to download and use, one can go to GeoGebra.org. GeoGebra is an up and coming dynamic teaching tool in our schools today. It is user-friendly for students and teachers. To connect geometry, measurement, and algebra to each other using the software, as students draw shapes, they can then measure them, and then derive formulas using the "Algebra View" to see such formulas. If math anxiety occurs frequently, then attention to effective teaching methods that overcome math anxiety are important. Teachers need to use best practices like incorporating technology like GeoGebra into math instruction.

GeoGebra can be used as a dynamic and interactive learning tool that will prepare students for future abstract mathematical concepts. When building a strong foundation in geometry, algebra, and measurement, primary-aged students find it fun and easy-to-use. Students may move from using the geoboards, to the virtual geoboards, then finally to GeoGebra. When the grid view is turned on, they can see GeoGebra as an extension of the geoboard. When elementary teachers use GeoGebra with students, it is suggested that they leave the grid on the screen and turn off the Algebra View tool to minimize student confusion.

In a globally competitive world where it is more important to prepare our students for the science, technology, engineering, and mathematics (STEM) fields, using this software covers all. It applies math, science, and engineering ideas while using and applying the technology. Sparks (2011) feels that as the STEM fields become more important for our students to study, our schools and teachers need to do more to address math anxiety so that our students are confident to study areas related to STEM.

PowerPoint and Data Files for GeoGebra as they relate to this presentation and paper can be accessed at: http://matharoundus.com/GeoGebra. See Appendices A, B, and C for additional resources and examples for using GeoGebra.

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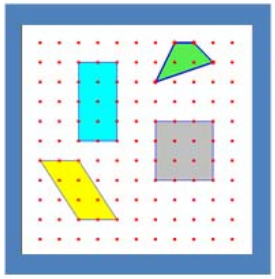
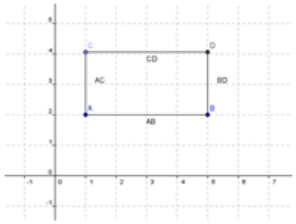
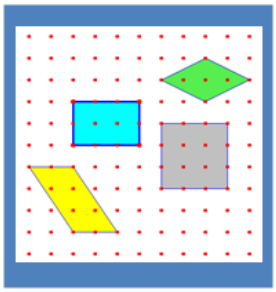
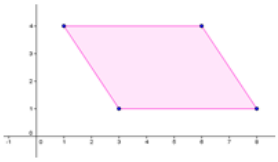
Appendix A: GeoGebra and Virtual Manipulative Websites and Resources for the Classroom

Web Sites	URL
National Library of Virtual Manipulatives	http://nlvm.usu.edu/en/nav/topic_t_3.html
National Council of Teachers of Mathematics	http://www.nctm.org/standards/content.aspx?id=25007
Cut the Knot	http://www.nctm.org/standards/content.aspx?id=25007
Geoboard Resources	http://msteacher.org/epubs/math/QuickTakes/geoBoard.aspx
GeoGebra	http://geoGebra.org
GeoGebra Wiki Forum	http://www.GeoGebra.org/en/wiki/index.php/Main_Page
GeoGebra Data Files	http://matharoundus.com/GeoGebra
Common Core Math Standards	http://www.corestandards.org/

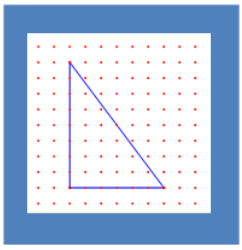
Appendix B: GeoGebra Resources (Examples from http://wiki.geogebra.org/en)

Math Concepts	URL
Addition of Integers	http://www.geogebra.org/en/upload/files/DoubleCross/integer_add.ggb
Addition of Fractions	http://www.geogebra.org/en/upload/files/Duke_22/Operations/addition_fraction.ggb
Venn Diagrams	http://www.geogebra.org/en/upload/files/Line/SetTheory.ggb
Area of a Rectangle	http://www.geogebra.org/en/upload/files/Duke_22/Length_Area_Volume/Rectangle to Trapezoid/Area_of_a_rectangle.ggb
Tangrams	http://www.geogebra.org/en/upload/files/xavi/tangram_armado.ggb

Appendix C: Sample Examples of GeoGebra Activities to teach Geometry, Measurement, and Algebra Ideas

Geometric Shape	Geometric Relationships
Geometry/Quadrilateral  **Definition of a Quadrilateral:** A closed four-sided figure made up of segments. **Link to GeoGebra:** Quadrilateral Copyright 2012 Carol A. Marinas, Joseph M. Furner	• Angles of a quadrilateral add to 360 degrees
Rectangle – Parallelogram with right angles  **Link to GeoGebra:** Rectangle Copyright 2012 Carol A. Marinas, Joseph M. Furner	• All angles have a measure of 90 degrees. • Opposite sides have the same measure. • Diagonals have the same measure. • Diagonals bisect each other.
Parallelogram/Measurement Quadrilateral with opposite sides parallel   **Link to GeoGebra:** Parallelogram Area of a Parallelogram Copyright 2012 Carol A. Marinas, Joseph M. Furner	• Opposite angles have the same measure. • Opposite sides have the same measure. • Adjacent angles are supplementary. • Area of parallelograms can be derived from the area of rectangles.

Right Triangle –
Triangle with a right angle



Link to GeoGebra: Right Triangle

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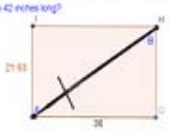
- Right angle is 90 degrees.
- Sum of acute angles are complementary.
- The Pythagorean Theorem is $a^2 + b^2 = c^2$.

Example Math Problem
Geometry, Measurement, & Algebra

Example :

On Japanese trains, there is a rule that forbids passengers bringing onto the train objects longer than 36 inches. How did a passenger travel with a ceremonial sword that was 42 inches long?



Change the slider to change the value of one side of the rectangle. The length of the other side will change accordingly. Place the sword inside the rectangle by moving point B.

Why are the lengths of the sides of the rectangle between 21.63 and 36?

Pythagorean Theorem

$a^2 + b^2 = c^2$

Uses Geometry, Measurement, and Algebra

sword.ggb file

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- Real-life example by inserting a picture.
- Show mathematical relationships with this example when the Pythagorean Theorem is used.