DISCOVER NUMBER THEORY IN GRADES K-12 USING THE SQUARE TOOL, AN INTERACTIVE JAVA TOOL

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Number Theory has fascinated people for thousands of years. The Pythagoreans explored many aspects of number theory as early as 550 BCE. Many aspects of number theory lead into mainline topics in the math classroom whether related to fractions, factors, multiples, or prime numbers. Along with such mainline topics covered today in schools, there are many other intriguing number patterns that can be explored and connected to what students learn in mathematics each day. Number patterns are useful for developing problem-solving skills. Recognizing number patterns allows students to think logically and apply critical thinking skills as they progress through primary and secondary education. The Annenberg Media (2006) suggest that number patterns help students learn how to count by 1s, 2s, 5s, and 10s. As they get older, students learn how to transition from sums to products. By high school, students are encouraged to use number patterns to understand functions and complex operations (Annenberg Media, 2006).

This paper discusses the observation of number patterns and number theory concepts using the Square Tool, an interactive java program (Su, Marinas, & Chraibi, 2008). Activities using the Square Tool will be shared in the K-12 classroom.

Number Theory is considered to be a recreational branch of mathematics for mathematicians (Reys, Lindquist, Lambdin, Smith and Suydam, 2009). Many find it intriguing to seek out number patterns and define such things while looking at factors of numbers. Things like abundant, amicable, composite, deficient, perfect, prime, and square numbers. Students first encounter number theory when studying mathematics. Some mathematicians spend their entire life exploring this field.

Wall (2010) contends that students should begin their study and work with number theory as early as elementary school. Reys, Lindquist, Lambdin, Smith and Suydam (2009) feel that it is very important to motivate the study of number relationships. They feel there are four main reasons number theory concepts need to be incorporated in our classrooms: (1) numbers can be fascinating, (2) number theory opens the doors to many mathematical conjectures, (3) number theory provides a way to extend and practice many math skills, and (4) number theory offers a source of recreation. The fourth reason is exciting, as it is really important to turn students on to math and include using number games and puzzles. By incorporating the Square Tool, students can see and explore many intriguing patterns while developing good number sense in math class.

As students learn and explore concepts in number theory, Pathak (2008) emphasizes how they are encouraged to make conjectures. For example, with the Square Tool, students may be asked to make a 7 x 7 grid and ask where the multiples of 7 are located, then ask them to make conjectures about what happens with another number multiple. Or by allowing them to explore with the diagonals, they can make conjectures about some interesting diagonal sums. As they interact with The Square Tool, students freely explore conjectures. Teachers need to allow students to explore and make conjectures in the learning of mathematics.

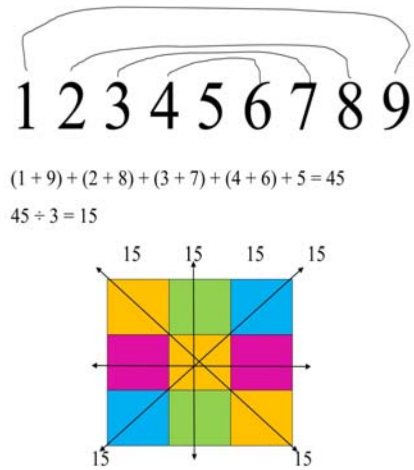
Technology is playing an increasingly important role in the teaching and learning of mathematics at all levels. NCTM (2005) feels that mathematical learning is strongly impacted by using technology so that students better understand mathematics in today's high-tech globally competitive world. Furner and Marinas (2007) found that young students are quickly learning and becoming eager to use the technology as it makes the learning of the material easier and more exciting. The Square Tool can serve as (1) a bridge connecting the concrete such as identifying factors by using color tiles to the abstract of the symbols only and (2) an emerging technology as it motivates and allows students to discover many intriguing number patterns while serving as a visual calculator. Sinclair, Zazkis, and Liljedahl (2004) have found in their work that issues related to the structure of natural numbers and the relationship among numbers are not well grasped by students. The authors describe a computer-based learning environment called "Number Worlds" that was designed to support the exploration of elementary number theory concepts by making the essential relationships and patterns more accessible to learners. Based on Zazkis and Liljedahl's research with pre-service elementary school teachers, they have found how both the visual representations embedded in the microworld and the possibilities afforded for experimentation affect learners' understanding and appreciation of basic concepts in elementary number theory. They also found that there were many positive aesthetic and affective dimensions of their participants' engagement with the learning environment.

The Square Tool is modeled after the hundred's chart which uses 10 rows and 10 columns to arrange the first 100 numbers. Various mathematical concepts will be introduced using the patterns found in the various number grids using the Square Tool.

The prime numbers in the Six, Seven and 13 Numbers Charts formed a diagonal that was obvious to recognize. Heinz (2006) observed that in a spiral square of numbers, at least one prime number was adjacent to $6n$ or multiples of six. But this is not true for every multiple of six. For instance, Heinz (2006) observed that the first exception is for $n=20$ where the adjacent numbers are 119, 120, and 121 which are all composite numbers. However, considering the Seven and 13 Numbers Charts in the Appendix, the multiples of six are running diagonal and adjacent to the diagonals of the primes. Referring back to the first observation about the Six Numbers Chart, it can be concluded that most primes can be located by using the formula $6n+1$ or $6n–1$.

A Sample Number Pattern: A magic square is a number arrangement with each number occurs once and the sum of the numbers of any row, any column, or any diagonal is the same. (Figure 1)

Figure 1: Magic Square



The Square Tool serves as a bridge which can be used to connect the concrete to the abstract ideas of number sense and number theory. The tool is a semi-concrete interactive means to help students make sense of number ideas.

In the case of a 9x9 square grid, the sum of either major diagonal is 369. While the Square Tool explores patterns, the tool has the ability to add numbers and interactively change square sizes.

In the elementary grades, the Square Tool adds distinct numbers and investigates patterns. At the elementary level, students are moving from hands-on activities to abstract numerical concepts promoting higher-level thinking skills. Students in grades 2 – 5, discovered that if you add two even numbers, the result is even. And adding two odd numbers will result with an even number. They also discovered that when you add an odd to an even number, the result is odd. A great investigative game is to allow students to add a combination of different numbers and predict the answer as odd or even. For example: students select an 8x8 grid, click on 1 and 8. The sum shows as 9. Reset, click on 1, 8, 9. This time the sum shows 18.

Sample Game: Number Pairs Game
The teacher clicks on a number pair and teams must identify another pair that has the same sum on the number grid shown. The commutative property can easily be shown here, for example, the teacher selects the numbers 3 and 9 and the students record the

sum or product. The teacher then clicks 9 and 3 and the students can see that the sum or product is still the same. This leads to the conjecture that 2 numbers can be added or multiplied in any order and the result is the same.

The Square Tool can be used to demonstrate that multiplication is based on **repeated addition**. The hands-on activities of groups of items help to develop concrete ideas about multiplication. If you want to multiply 8 x 2, make an 8x8 grid. Each row represents a group of 8 items. We want 2 groups (rows), so the answer is the last number in row 2, which is 16. For example, find 8 x 7 with the 8x8 grid. The last number in row 8 is _?__. What if the problem is **3** x **9**? Obviously, the **3x3** grid will not work here. Since multiplication is also commutative, create the grid for the larger number instead (the **9x9** grid). The Tool will also help the students conclude that division is the opposite of multiplication. Let's try 45 ÷ 9. Make a **9x9** grid. Note that 45 is in the last column of the 5^{th} row. That means that the answer is 5. Because it is in the last column, 9 is a multiple of 45. What about 52 ÷ 9? Using the same square, 52 is in the 6^{th} row, 7^{th} column. There are 5 complete rows and 7^{th} column. Answer: 5 remainder 7. What about 33 ÷ 7? Make a **7x7** grid, 33 is in the 5^{th} row, 5^{th} column. There are 4 complete rows and 5^{th} column. Answer: 4 remainder 5. This can lead to 33 = 7 × 4 + 5. And, 45 = 7 × ___+ ___. By filling in the blanks with different numbers, students build a foundation for Algebra.

In the middle schools, teachers can utilize the Square Tool to help introduce, reinforce, or teach concepts such as: number theory, prime and composite Numbers, Sieve of Eratosthenes, multiples, factors, amicable numbers, and Napier Bone arrays for lattice multiplication.

Number Theory is the study of the properties of integers. Mathematicians have been fascinated by number patterns of primes, factors, and amicable numbers. Euler, Gauss, Fermat, Euclid, and Pythagoras are some the important contributors to number theory. Important topics include prime and composite numbers, multiples, numeric relationships, and factors.

Teachers might want to use the Eratosthenes' Sieve JavaScript located at http://www.hbmeyer.de/eratosiv.htm to help students identify prime and composite numbers immediately. While this latter website is easier, it is best to have students use the Square Tool to click on the multiples and identify all the remaining numbers as primes. This will help promote understanding of the mathematical concepts of primes/composites and multiples.

Below is an example of the algorithm using the Square Tool:

1. Make a 10x10 grid.
2. Highlight all the multiples of two (4, 6, 8 etc.).
3. The next non-highlighted number is a prime number.
4. Highlight all multiples of the prime number from previous step.

5. Repeat steps 3 and 4 until you reach a number that is greater than the **square root** of 100 or 10 (the highest number in the square).
6. All the remaining numbers in the list are prime except 1 that is neither prime nor composite.

Other resources such as the Chartworld, by Don Ploger, is based on the Boxer Program, allows you to color multiples, all multiples of 2 for example yellow and all multiples of 3 blue, then making multiples of 6 in other color.

Another great resource is Shodor's Interactive Coloring Multiples in Pascal's Triangle-Multiple Game at:
http://www.shodor.org/interactivate/activities/ColoringMultiples/?version=1.6.0_11&browser=Mozilla&vendor=Sun_Microsystems_Inc

Teachers may use Color Tiles first to show factors of a number as a concrete example. Then, use the Square Tool to further reinforce the concepts. The following math concepts can be taught using the Square Tool.

- Identify factors of 6, 8, 12, 15 on a 4x4 square grid
- GCF and Proper Factors **-** GCF (Greatest Common Factor) is the largest factor that divides two numbers. GCF (18, 24) = 6
- **Proper Factors** are factors of *n* other than itself. The proper factors of 12 are {1, 2, 3, 4, 6}.
- **Abundant Numbers**: A number is abundant if the sum of its proper factors is greater than the number itself. For example, the proper factors of 24 are {1, 2, 3, 4, 6, 8, 12} and 1 + 2 + 3 + 4 + 6 + 8 + 12 = 36, so 24 is abundant.
- **Deficient Numbers**: A number is deficient if the sum of its proper factors is less than the number itself. For example, the proper factors of 14 are {1, 2, 7} and 1 + 2 + 7 = 10, so 14 is deficient.
- **Perfect Numbers**: A perfect number is a number that equals the sum of its proper factors. For example, the proper factors of 28 are {1, 2, 4, 7, 14} and 1 + 2 + 4 + 7 + 14 = 28, so 28 is perfect.
- **Amicable numbers** were known to the Pythagoreans, who credited them with many mystical properties. **Amicable numbers** are two different numbers so related that the sum of the proper divisors (factors) of one of the numbers is equal to the other. (A **proper divisor** of a number is a divisor other than the number itself.) For example, the smallest pair of amicable numbers is (220, 284); for the proper divisors of 220 are 1, 2, 4, 5, 10, 11, 20, 22, 44, 55 and 110, of which the sum is 284; and the proper divisors of 284 are 1, 2, 4, 71, and 142, of which the sum is 220. The first few amicable pairs are: (220, 284), (1184, 1210), (2620, 2924), (5020, 5564), (6232, 6368).

The Pythagoreans believed that amicable numbers, like all special numbers, had a profound cosmic significance. A biblical reference (a gift of 220 goats from Jacob to Esau, Genesis 23: 14) is thought by some to indicate an earlier knowledge of amicable numbers (http://science.jrank.org/pages/282/Amicable-Numbers.html). They are fun to explore!

High school students can use the Square Tool to explore modulus, adding all the numbers from 1 to 49 and then extend it to 1 to 100, and investigate Inner Square Relationships and can extend to proofs.

Below are examples of using the Square Tool to explore modulus.

- Make a 6x6 Grid to help understand mod 6.
- All the families with the same remainder will be down the same column.
- The "mod" operator in computer languages is simply the remainder.
  For example, **26** mod **6** = **2** because 26 / **6** = **4** rem **2** which in turn means 26 = **6** * **4** + **2**
  So in mod 6: 8, 14, 20, 26, 32 - all have a remainder of **2** and are in column **2**.

Below are examples to Add Numbers in Pairs:

- Numbers in pairs 1 + 49, 2 + 48, 3 + 47, etc. Note that each sum is 50.
- The sum of the numbers from 1 to 49 is (24 pairs of 50) + 25. **So that sum is 1225.**
- This can be extended to adding the first 100 numbers by making a 10x10 grid. The sum of each pair would be 101. Because it is an "even" square grid, it is **50 pairs of 101 for a sum of 5050.**

Conclusion

Number Theory encompasses many mainline concepts in the K-12 math curriculum. Teachers can teach such concepts while allowing students to discover many intriguing number patterns using technology. Exploring number patterns are an effective means for teaching various mathematical concepts. Although the Hundreds Chart is a commonly used in the classrooms, students can also benefit from analyzing various number patterns using other resources, such as the Square Tool. Even though the Square Tool has limitations when used as a calculator for addition, it provides a visual representation of the addends. While exploring different number theories as primes and composite numbers, it functions as a great investigative tool for students. These early concepts led to deeper number theories in the middle and high schools. Although the Hundreds Chart does not display the obvious patterns found for primes and multiples of 6 as the 7 and the 13, it is very useful for teaching multiples of 5 and 10. Working with number charts or the Square Tool can help students understand the relationship between numbers and evaluate the effects of the operations used to manipulate them. Turning students on to Number Theory and exploring all the intriguing number patterns that exist in the mathematics world while using technology, like the Square Tool, can bode well for promoting mathematics and creating the next generation of number theorists in our world as well. Hopefully, the topic of number theory, the exploration of patterns, and the technology of the Square Tool will motivate our students mathematically.

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Appendix

The shaded numbers in each number pattern represent prime numbers.

Figure 2: Ten Numbers Chart

1	**2**	**3**	4	**5**	6	**7**	8	9	10
11	12	**13**	14	15	16	**17**	18	**19**	20
21	22	**23**	24	25	26	27	28	**29**	30
31	32	33	34	35	36	**37**	38	39	40
41	42	**43**	44	45	46	47	48	49	50
51	52	**53**	54	55	56	57	58	**59**	60
61	62	63	64	65	66	**67**	68	69	70
71	72	**73**	74	75	76	77	78	**79**	80
81	82	**83**	84	85	86	87	88	**89**	90
91	92	93	94	95	96	**97**	98	99	100

Figure 3: Six Numbers Chart

1	**2**	**3**	4	**5**	6
7	8	9	10	**11**	12
13	14	15	16	**17**	18
19	20	21	22	**23**	24
25	26	27	28	**29**	30
31	32	33	34	35	36
37	38	39	40	**41**	42
43	44	45	46	**47**	48
49	50	51	52	**53**	54
55	56	57	58	**59**	60
61	62	63	64	65	66
67	68	69	70	**71**	72
73	74	75	76	77	78
79	80	81	82	**83**	84
85	86	87	88	89	90
91	92	93	94	95	96
97	98	99	100	**101**	102

Figure 4: Seven Numbers Chart

1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30	31	32	33	34	35
36	37	38	39	40	41	42
43	44	45	46	47	48	49
50	51	52	53	54	55	56
57	58	59	60	61	62	63
64	65	66	67	68	69	70
71	72	73	74	75	76	77
78	79	80	81	82	83	84
85	86	87	88	89	90	91
92	93	94	95	96	97	98
99	100	101	102	103	104	105

Figure 5: 13 Numbers Chart

1	2	3	4	5	6	7	8	9	10	11	12	13
14	15	16	17	18	19	20	21	22	23	24	25	26
27	28	29	30	31	32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47	48	49	50	51	52
53	54	55	56	57	58	59	60	61	62	63	64	65
66	67	68	69	70	71	72	73	74	75	76	77	78
79	80	81	82	83	84	85	86	87	88	89	90	91
92	93	94	95	96	97	98	99	100	101	102	103	104

Figure 6: 21 Numbers Chart

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42
43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63
64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84
85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100	101	102	103	104	105