USING AN INTERACTIVE JAVA TOOL TO EXPLORE NUMERIC RELATIONSHIPS FOR ALL GRADE LEVELS

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Mathematicians and historians affirm that magic squares are the remains of the ancient China around 2200 B.C. The magic square was associated in the square that appeared on the shell of a sacred turtle. According to the Chinese myth, while Emperor Yu was walking along the Yellow River, he noticed a unique diagram on the back of the tortoise shell. The Emperor called the numerical pattern *lo shu*. This is the earliest magic square recorded in the first-century book *Da-Dai Liji*. (Anderson, 2001). Using a Java Square Tool to show intriguing number patterns for all grade levels K-12, classroom teachers will find this tool useful in making connections and bridging from the concrete to the abstract.

Example of a Magic Square and a Square Tool

A magic square is a number arrangement with each number occurs once and the sum of the numbers of any row, any column, or any diagonal is the same. In Figure 1, the sum for the Magic Square is 111.

Figure 1: Magic Square

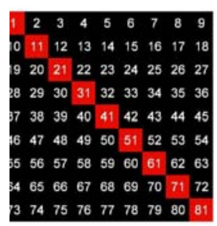

Figure 2: Square Tool

In Figure 2, the table above is made of squares of consecutive numbers as shown made with the interactive Square Tool. Only the diagonals of these squares will have the same sum in this case, 369.

The Square Tool

The Square Tool is a Java-supported software which can investigate patterns allowing the teachers to explore infinite possibilities of venues in which to teach children. It can promote one's ability to recognize similarities quickly and correctly, apply patterns to different subjects and venues, and form the foundation of fundamental patterns, such as counting by tens. The Square Tool can be used for any grade level and supports the current math curriculum in most states and countries and the National Council of Teachers of Mathematics (NCTM) *Standards*. Furner (2005) and Ozel, Yetkiner, and Capraro (2008) discusses the importance of using technology in the classroom and making connections by bridging from the concrete to the abstract using such emerging technologies and societies reliance on such technologies. The Square Tool can serve as such a tool for helping students discover many intriguing number patterns and ideas.

Elementary Grades

At the elementary level, students are moving from hands-on activities to abstract numerical concepts. The Square Tool adds distinct numbers and investigates patterns and therefore promotes higher-level thinking skills. The Square Tool can be used to perform operations and show numeric relationships such as addition, multiplication, division, and various number properties.

Numeric Relationships: Add two even numbers, the result is even. Add two odd numbers, the result is even. Add an odd to an even number, the result is odd.

Addition: Using the Square Tool, find a pattern with the sum of 2 numbers that add to 9.

Multiplication: Multiplication is based on repeated adding of groups. The hands-on activities of groups of items help to develop concrete ideas about multiplication. The Square Tool leads to a more abstract view of multiplication using numbers. If you want to multiply 8 x 2, make a 8x8 square. Each row represents a group of 8 items. We want 2 groups (rows), so the answer is the last number in row 2 or 16.

Division: Since division is the opposite of multiplication, the tool will help explore this relationship. Let's try $33 \div 7$. Make a 7x7 square (Figure 3), 33 is in the 5th row, 5th column. There are 4 complete rows and 5th column. Answer: 4 remainder 5. This can lead to $33 = 7 \times 4 + 5$. This can lead to $45 = 7 \times \square + \square$. By filling in the boxes with different numbers, students build a foundation for Algebra.

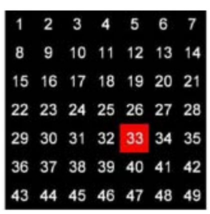

Figure 3: Division

Figure 4: Sieve of Eratosthenes

Middle School Grades

In the middle grades, the application of the Square Tool is useful with number theory, prime and composite numbers, the Sieve of Eratosthenes, multiples, factors, and amicable numbers. Number Theory is the study of the properties of integers. Mathematicians such as Euler, Gauss, Fermat, Euclid, and Pythagoras have been fascinated by number patterns of primes, factors, and amicable numbers and provided important contributions to number theory. Important topics in this area are prime and composite numbers, multiples, numeric relationships, and factors. Wall (2010) feels that number theory ideas should be introduced in the elementary grades and that classroom teachers should have a sound understanding of basic ideas.

Prime and Composite Numbers: While Gauss was considered the Father of Primes and Composites Exploration, Erathosthenes created a Sieve to find the prime numbers in an organized fashion (Figure 4).

The Algorithm of the Sieve of Erathosthenes: (Using the Square Tool)

- Make a 10x10 square.
- Highlight all the multiples of two (4, 6, 8 etc.).
- The next non-highlighted number is a prime number.
- Highlight all multiples of the prime number from previous step.
- Repeat steps 3 and 4 until you reach a number that is greater than the **square root** of 100 or 10 (the highest number in the square).
- All the remaining numbers in the list are prime except 1 that is neither prime nor composite.

While this next web site is easier, it is best to have students use the Square Tool to click on the multiples and identify all the remaining numbers as primes. If done in this order, it will promote understanding of the mathematical concepts of primes/composites and multiples. Eratosthenes' Sieve JavaScript at: http://www.hbmeyer.de/eratosiv.htm

Multiples: Multiples can be explore via concrete examples to interactive web sites.

- Color Tiles as concrete examples
- Using the Square Tool and making differing square sizes, discuss the multiples of the square size are found in the last column.
- Chartworld, by Don Ploger, based on the Boxer Program, allows you to color multiples, all multiples of 2 for example yellow and all multiples of 3 blue, then making multiples of 6 green.
- Shodor's Interactive Coloring Multiples in Pascal's Triangle-Multiple Game at: http://www.shodor.org/interactivate/activities/ColoringMultiples/?version=1.6.0_11&browser=Mozilla&vendor=Sun_Microsystems_Inc

Factors: Use the Square Tool to identify factors of 6, 8, 12, 15 on a 4x4 square grid.

GCF and Proper Factors:

- **GCF (Greatest Common Factor)** is the largest factor that divides two numbers. GCF (18, 24) = 6
- **Proper Factors** are factors of *n* other than itself.
  The proper factors of 12 are {1,2 , 3, 4, 6}.

Abundant Numbers, Deficient Numbers, Perfect Numbers:

- **Abundant Numbers**: A number is abundant if the sum of its proper factors is greater than the number itself. For example, the proper factors of 24 are {1, 2, 3, 4, 6, 8, 12} and 1 + 2 + 3 + 4 + 6 + 8 + 12 = 36, so 24 is abundant.
- **Deficient Numbers**: A number is deficient if the sum of its proper factors is less than then number itself. For example, the proper factors of 14 are {1, 2, 7} and 1 + 2 + 7 = 10, so 14 is deficient.
- **Perfect Numbers**: A perfect number is a number that equals the sum of its proper factors. For example, the proper factors of 28 are {1, 2, 4, 7, 14} and 1 + 2 + 4 + 7 + 14 = 28, so 28 is perfect.

Amicable Numbers:

- The Pythagoreans credited **amicable numbers** with mystical properties. **Amicable numbers** are two different numbers so related that the sum of the proper factors of one of the numbers is equal to the other.
- The smallest pair of amicable numbers is (220, 284); for the proper factors of 220 are 1, 2, 4, 5, 10, 11, 20, 22, 44, 55 and 110, of which the sum is 284; and the

proper factors of 284 are 1, 2, 4, 71, and 142, of which the sum is 220.

- The first few amicable pairs are: (220, 284), (1184, 1210), and (2620, 2924).

High School Grades

Figure 5: Mod 6

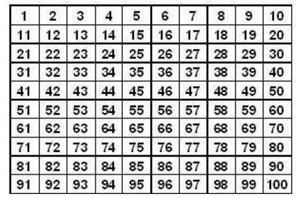

Figure 6: 10 x 10 Square

Modulus: The "mod" operator in computer languages is simply the remainder. Using the Square Tool, make a 6x6 Square to help understand mod 6 (Figure 5). All the families with the same remainder will be down the same column. For example, **26** mod **6** = **2** because 26 / **6** = **4** remainder **2** which in turn means 26 = **6** * **4** + **2.** In **mod 6:** 8, 14, 20, 26, 32, all have a remainder of **2** and are in column **2**.

Hundreds Chart: Based on reviewing the hundreds square which arranges the first 100 numbers respectively into 10 rows and 10 columns (Figure 6), we will look for patterns and draw appropriate conjectures.

Observed Patterns with Diagonals: Utilizing the hundred's chart as a base, the following number patterns can be discovered using any size square. In this example of a 6x6 square:

Figure 7: Corners

Figure 8: Sum of Diagonals

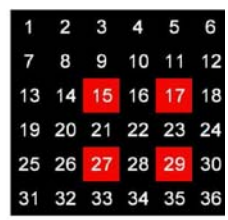

Figure 9: Inner Boxes

1. Numbers in the opposite ends of diagonals all add up to 37, for example in Figure 7: 1 + 36 = 37 **or** 6 + 31 = 37 **or** 26 + 11 = 37 **or** 15 + 22 = 37
2. In Figure 8, the sum of the major diagonals add to 111. 1 + 8 + 15 + 22 + 29 + 36 **or** 6 + 11 + 16 + 21 + 26 + 31
3. If you take an "n x n" box like in Figure 9, the diagonal numbers at the each corner add to the same number. For example: In the 2 x 2 box at the top corner 1 + 8 = 9 and 7 + 2 = 9. In the 3 x 3 box, 15 + 29 = 44 and 27 + 17 = 44.

Internal Square Relationships: In the 3x3 internal square, 9 + 21 + 14 + 16 = 60 (Figure 10) and the diagonal sums each equal 75 (Figure 11). The ratio 60/75 is 4/5 as summarized in the table below. Students can explore other possibilities and even using calendars.

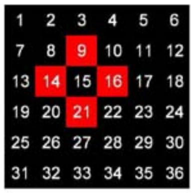

Figure 10: Remaining Sum

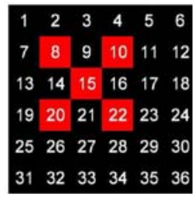

Figure 11: Diagonal Sum

	n x n Number Table
Internal 3 x 3	Ratio $\frac{\text{Remaining Sum}}{\text{Diagonal Sum}} = \frac{4}{5}$
Internal 4 x 4	Ratio $\frac{\text{Remaining Sum}}{\text{Diagonal Sum}} = 1$
Internal 5 x 5	Ratio $\frac{\text{Remaining Sum}}{\text{Diagonal Sum}} = \frac{16}{9}$

Summary

The Square Tool is an emerging technology that is useful in showing intriguing number patterns for all grade levels K-12. Classroom teachers will find this tool useful in making connections and bridging the gap from the concrete to the abstract. The Square Tool has relationships to magic squares which have fascinated people for over 2000 years. The Square Tool can be useful in showing basic addition and division facts, to primes, multiples, and number theory ideas to even modulus at the high school level. Again, as Ozel, Yetkiner, and Capraro (2008) conclude, using technology in the math classrooms supports different teaching and learning strategies and objectives. Young people can use the Square Tool as a semi-concrete bridge to discover many number concepts. The Square Tool can be found at: http://matharoundus.com/ICTCM.

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