EXPLORING NUMERICAL RELATIONSHIPS THROUGH INTERACTIVE NUMBERED SQUARES OF DIFFERING SIZES

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Abstract:
Through this interactive java tool found at the web site below, users will be able to explore various mathematics relationships found in numbered squares. These relationships include multiples, factors, ratios of comparative numbers, and other mathematics content areas.

Magic Square Tool (Interactive Java Calculation Tool):
http://mcs-research.barry.edu/squares/index.html

Exploring Number Patterns with a Numbered Square

The base 10 system of counting numbers commonly used today uses 10 digits. The design of a 100's table that has all of the digits 0-9 on the first line can illustrate many important mathematical concepts such as, place value, multiples, multiple patterns, least common multiples, and more. One example where the hundreds square comes in handy is teaching the concept of '0'. Often '0' is not used as a number alone, but as a placeholder. It can be argued that zero is more valuable than the other numbers; because zeros used as a place holder allows our number system to be based on place value, enabling us to represent all numbers using only the 10 digits of our number system. Zero helps us assign value to numbers and thus, is extremely valuable.

In looking for patterns that exists on a hundreds square, we find infinite possibilities of venues in which to teach children. Inherent therein is one of the most fundamental patterns, which is counting by tens. The basis for teaching patterns is promotion of one's ability to recognize similarities quickly and correctly and being able to apply one pattern to a different venue and even different subjects.

Based on reviewing the hundred's square, which arranges the first 100 numbers respectively into 10 rows and 10 columns. We will investigate squares inside a 6, 7, and 13 numbered squares. In addition, we will show how students can take any numbered square and make conjectures about its internal 2 x 2 and 4 x 4 boxes (squares). For each of the numbers, we will also look for patterns and draw appropriate conjectures.

The hundreds square arranges the first 100 numbers into 10 rows and 10 columns. Utilizing the hundreds square as a base, the following number patterns can be discovered:

Table 1
10 x 10 Numbers are arranged into 10 rows and 10 columns

1	**2**	3	4	5	6	7	8	9	10
11	**12**	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	**35**	**36**	**37**	38	39	40
41	42	43	44	**45**	**46**	**47**	48	49	50
51	52	53	54	**55**	**56**	**57**	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Observed patterns:

1. Numbers in the opposite ends of diagonals all add up to 101, for example: **1 + 100 = 101, 10 + 91 = 101, 12 + 89 = 101, or 19 + 82 = 101.**

2. The sum of the major diagonals adds to 505 (1+12+23+34+45+56+67+78+89+100 or 10+19+28+37+46+55+64+73+82+91).

3. In the below pattern, if you take an "n x n" box, the diagonal numbers at the each corner add to the same number. For example, let's take a 2 x 2 box at the top corner shown in pink 1 + 12 = 13 and 2 + 11 = 13. Let's take a 3 x 3 box shown in orange, 35 + 57 = 92 and 37 + 55 = 92.

6 X 6 SQUARE

Table 2
6 x 6 Numbers are arranged into 6 rows and 6 columns

1	**2**	3	4	5	6
7	**8**	9	10	11	12
13	14	**15**	**16**	**17**	18
19	20	**21**	**22**	**23**	24
25	26	**27**	**28**	**29**	30
31	32	33	34	35	36

Observed patterns:

1. Numbers in the opposite ends of diagonals all add up to 37, for example:

1 + 36 = 37, 6 + 31 = 37, 26 + 11 = 37, or 15 + 22 = 37.

2. The sum of the major diagonals adds to 111 (1+8+15+22+29+36 **or** 6+11+16+21+26+31).

3. In the pattern below, if you take an "n x n" box, the diagonal numbers at the each corner add to the same number. For example, let's take a 2 x 2 box at the top corner shown in pink 1 + 8 = 9 and 7 + 2 = 9. Let's take a 3 x 3 box shown in orange, 15 + 29 = 44 and 27 + 17 = 44.

7 x 7 SQUARE

Table 3
7 x 7 Numbers are arranged into 7 rows and 7 columns

1	**2**	3	4	5	6	7
8	**9**	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	**25**	**26**	**27**	28
29	30	31	**32**	**33**	**34**	35
36	37	38	**39**	**40**	**41**	42
43	44	45	46	47	48	49

Observed patterns:

1. Numbers in the opposite ends of diagonals all add up to 50, for example:

   49 + 1 = 50, 37 + 13 = 50, 17 + 33 = 50.

2. Major diagonals add to 175 (1+9+17+25+33+41+49 or 7+13+19+25+31+37+43).

3. In the below pattern, if you take an "n x n" box, the diagonal numbers at the each corner add to the same number. For example, let's take a 2 x 2 box at the top corner shown in pink 1 + 9 = 10 and 8 + 2 = 10. Let's take a 3 x 3 box shown in orange, 25 + 41 = 66 and 39 + 27 = 66

4. Prime numbers fall on diagonals, such as 5, 11, 17, 23, and 29.

<u>13 x 13 SQUARE</u>

Table 4
13 x 13 Numbers are arranged into 13 rows and 13 columns

1	2	3	4	5	6	7	8	9	10	11	12	13
14	15	16	17	18	19	20	21	22	23	24	25	26
27	28	29	30	31	32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47	48	49	50	51	52
53	54	55	56	57	58	59	60	61	62	63	64	65
66	67	68	69	70	71	72	73	74	75	76	77	78
79	80	81	82	83	84	85	86	87	88	89	90	91
92	93	94	95	96	97	98	99	100	101	102	103	104
105	106	107	108	109	110	111	112	113	114	115	116	117
118	119	120	121	122	123	124	125	126	127	128	129	130
131	132	133	134	135	136	137	138	139	140	141	142	143
144	145	146	147	148	149	150	151	152	153	154	155	156
157	158	159	160	161	162	163	164	165	166	167	168	169

Observed patterns:

1. Numbers in the opposite ends of diagonals all add up to 170, for example **169 + 1 = 170, or 157 + 13 = 170**. As seen in the above diagram.

2. In the above pattern, if you take an "n x n" box, the diagonal numbers at each corner add to the same number. For example let's take a 2 x 2 box at the bottom left hand corner shown in pink 132 + 120 = 252 and 119 + 133 = 252. Let's take a 3 x 3 box shown in orange, 114 + 142 = 256 and 116 + 140 = 256

3. Prime numbers fall on the selected diagonals, such as 5, 17, 29, 41, not true for the entire 13 x 13 pattern.

Summary of patterns:

Students are offered many opportunities to discover various mathematical concepts through the use of patterns. For example, division can be taught by using numbered squares. In a 6 x 6 numbered square, 44 ÷ 6 can be found by locating 44. There are 7 complete rows above 44 and 2 numbers left in row 8. This would be interpreted to 44 ÷ 6 is 7 with a remainder of 2.

With the development of puzzles, games, and computer programs, students have a chance to learn and practice the skills that are being developed within early mathematic classes. As one looks closer to the patterns that exist within different number arrangements, it becomes obvious that the patterns are constant among them. This is shown throughout all the different arrangements of the different number patterns shown here in this paper.

In all the patterns, if you take an "n x n" box, the numbers in the diagonal boxes add up to the same number. For example, let's take a 2 x 2 box in the 21 x 21 square, at the bottom left hand corner shown in orange 192 + 214 = 406 and 193 + 213 = 406. Let's take a 4 x 4 box shown in green, 199 + 139 = 338 and 136 + 202 = 338.

In this exercise as one looks closer to different patterns, it seems more and more commonalities exist between them. Sometimes it can be good to get the students out of their comfort zone of base 10 number arrangements. It's like shaking hands using the left hand verses the right; it will make a lasting expression and encourage learning. Different number patterns may intrigue a student's sense of discovery; it can also be used as a teaching tool. Our number system, built on the base–ten pattern, works very well and has been shown to be very efficient. It should be noted that there are more patterns that exist within the base 10 system than any of the patterns we have looked at in this paper. In the base 10 system, addition and subtraction is made easy using various techniques such as the "best friend"[1] sets; 1 + 9, 2 + 8, 3 + 7, 5 + 5, 4 + 6, 0 + 10, etc. Numbers are arranged in rows and columns very symmetrically. Our personal preference is to use the base 10 number system for student's rudimentary mathematics skills, as it provides a clearer and more advantageous set of patterns when compared to the different base patterns depicted here in this paper.

References

Drexel University. *The Math Forum: Ask Dr. Math FAQ.* <http://mathforum.org/dr.math/faq/faq.order.operations.html> (24 January 2005).

Drexel University. *Patterns and Sequences.* <http://mathforum.org/library/problems/topics/prime_numbers/> (26 January 2005).

Earlham University. *Are all primes (past 2 and 3) of the forms 6n+1 and 6n-1?* <http://www.earlham.edu/~rodrimi/primes_multiple_of_6.htm> (25 January 2005).

Smith, Karl J. *The nature of mathematics*. (10th ed.). Pacific Grove, CA: Thomson Leaning, 2003.

Su, H.F. and T.C. Su. *From arithmetic to algebra: An interdisciplinary approach to teaching pre K through 8th grade mathematics.* Boston, MA: Houghton Mifflin Co., 2004.

[1] Su, H.F., Su, T.C. (2004) *From arithmetic to algebra: An interdisciplinary approach to teaching pre K through 8th grade mathematics.* Houghton Mifflin Co., Boston, MA.