Exploring Relationship of Number Patterns Using Number Tables

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Abstract:

Through this study of square number tables, users will be able to explore various mathematics relationships like multiples and prime numbers. These relationships will lead to a study of Pascal's Triangle. This paper helps to strengthen numeric relations to provide a foundation for Number Theory. Four activities are provided for student growth and web sites are included for further extended study.

Key Words: Multiples, Number Theory, Pascal's Triangle, Prime Numbers

Exploring Relationship of Number Patterns
Using Number Tables

The base 10 system of counting numbers commonly used today uses 10 digits. These digits are 1, 2, 3, 4, 5, 6, 7, 8, 9, and 0. Although we do not use zero in counting, it does help us assign place value to numbers and thus, is extremely valuable. In looking for patterns that exist on the Hundred Board, we find infinite possibilities of venues in which to teach children. Inherent therein is one of the most fundamental patterns, which is counting by tens. The tens pattern is very vital in teaching multiplication, addition, subtraction, time and money. The basis for teaching patterns is the enhancement of one's ability to recognize similarities quickly and correctly and being able to apply one pattern to multiple venues and even multiple subjects.

When arranging the hundreds board by 6's instead of 10's, other patterns are discovered. A number in the second row is 6 more than the number directly above it (See Table 1).

1	2	3	4	5	6
7	8	9	10	11	12

Table 1
6 x 6 Numbers

When an instructor is teaching young students to add, this method can be used to show them how adding a constant affects change. Another evident pattern is the numbers in the last column are multiples of 6.

By adding more rows to the 6's board, a more explicit pattern happens in the 6^{th} row. The digit in the one's place in the 6^{th} row is the same as the digit written in the first row (See Table 2). When we continued to the seventh row, the digit in the one's place was the same as those in the second row.

1	2	3	4	5	6
7	8	9	10	11	12
13	14	15	16	17	18
19	20	21	22	23	24
25	26	27	28	29	30
31	32	33	34	35	36
37	38	39	40	41	42

Table 2
6 x 6 Numbers

When re-constructing the board by sevens, the initial observation is that the numbers in the last column are all multiples of 7 (See Table 3). When looking down each column,

the pattern of even odd even odd occurs. Moving diagonally to the right and downward, each number increases by 8. For example, stepping from 4 diagonally to 12 is a difference of 8. Another pattern is the ending number in row 4 is half the ending number in row 8.

1	**2**	**3**	**4**	**5**	**6**	**7**
8	**9**	**10**	**11**	**12**	**13**	**14**
15	**16**	**17**	**18**	**19**	**20**	**21**
22	**23**	**24**	**25**	**26**	**27**	**28**
29	**30**	**31**	**32**	**33**	**34**	**35**
36	**37**	**38**	**39**	**40**	**41**	**42**
43	**44**	**45**	**46**	**47**	**48**	**49**
50	**51**	**52**	**53**	**54**	**55**	**56**
57	**58**	**59**	**60**	**61**	**62**	**63**
64	**65**	**66**	**67**	**68**	**69**	**70**
71	**72**	**73**	**74**	**75**	**76**	**77**

Table 3
7 x 7 Numbers

In the first 10 rows of this pattern, no column has a digit in the ones place which is the same to any other number in that column. In the 11^{th} row, the digit in the ones place is the same as the digits in row 1.

The number charts provide many learning opportunities to discover mathematical patterns. For example, looking at a chart with the 21's pattern, the ending number of each row is a multiple of 21; thus, they are also multiples of 3 and 7. You might notice that vertically, the digit in the ones place increases by one.

In considering the above number patterns and through further investigations with the activities provided here, the foundation of basic number theory can be achieved. Students can also apply the strategies in solving the tables to other areas of mathematics such as the Pascal's Triangle and hence prepare students for the higher mathematical concepts. For example, Pascal's Triangle can be used to determine the coefficients of the binomial expansion:

$$(x + y)^0 = \mathbf{1}(x + y)^1 = \mathbf{1}x + \mathbf{1}y$$
$$(x + y)^2 = \mathbf{1}x^2 + \mathbf{2}xy + \mathbf{1}y^2$$
$$(x + y)^3 = \mathbf{1}x^3 + \mathbf{3}x^2y + \mathbf{3}xy^2 + \mathbf{1}y^3$$
$$(x + y)^4 = \mathbf{1}x^4 + \mathbf{4}x^3y + \mathbf{6}x^2y^2 + \mathbf{4}xy^3 + \mathbf{1}y^4$$

Understanding the binomial expansion can help students calculate enormous powers such as 2^{17}.

Below is an illustration of a simpler problem:

$2^5 = (1 + 1)^5$

$= 1(1)\ (1) + 5\ (1)\ (1) + 10\ (1)\ (1) + 10\ (1)\ (1) + 5\ (1)\ (1) + 1\ (1)\ (1)$

$= 32$

Have fun with your students exploring number patterns and relationships with the following pages of activities!

More fun with Pascal's Triangle can be found at the following websites (Needed for the Pascal's Triangle Handout):

- http://nlvm.usu.edu/en/nav/frames_asid_181_g_4_t_1.html?open=activities
- http://mathforum.org/dr.cgi/pascal.cgi?rows=10
- http://mathforum.org/dr.math/faq/faq.pascal.triangle.html
- http://mathforum.org/dr.math/faq/faq.comb.perm.html
- http://dimacs.rutgers.edu/~judyann/LP/lessons/12.days.pascal.html
- http://www.ualr.edu/lasmoller/pascalstriangle.html
- http://milan.milanovic.org/math/english/contents.html
- http://educ.queensu.ca/~fmc/december2001/Pascal.htm
- http://ptri1.tripod.com/

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Exploring Different Number Tables

ACTIVITY ONE: 6 x 6 Table

1	**2**	**3**	**4**	**5**	**6**
7	**8**	**9**	**10**	**11**	**12**

Table 1
6 x 6 Numbers are arranged into 6 rows and 6 columns

1. Complete the table.

2. What do you notice about the numbers in the 6^{th} column?

3. What about the numbers in the 2^{nd}, 4^{th}, and 6^{th} columns?

4. What about the numbers in the 1^{st}, 3^{rd}, and 5^{th} columns?

5. What about the numbers in the 3^{rd} and 6^{th} columns?

6. Let's look at the major diagonal, starting at 1 in the upper left corner and moving to the lower right corner. What relationship do those numbers have?

7. Let's look at the major diagonal, starting at 31 in the lower left corner and moving to the upper right corner. What relationship do those numbers have?

8. What about the relationships of the minor diagonals? Such as moving downward from 2 to 9 to 16 to 23 to 30? How about moving upward from 32 to 27 to 22 to 17 to 12? Do those relationships work with all minor diagonals?

ACTIVITY TWO: 7 x 7 Table

1	**2**	**3**	**4**	**5**	**6**	**7**
8	**9**	**10**	**11**			

Table 2
7 x 7 Numbers are arranged into 7 rows and 7 columns

1. Complete the table.

2. Let's look at the major diagonal, starting at 1 in the upper left corner and moving to the lower right corner. What relationship do those numbers have?

3. Let's look at the major diagonal, starting at 43 in the lower left corner and moving to the upper right corner. What relationship do those numbers have?

4. What about the relationships of the minor diagonals? Such as moving downward from 2 to 10 to 18 to 26 to 34 to 42? How about moving upward from 29 to 23 to 17 to 11 to 5? Do those relationships work with all minor diagonals?

5. Find the sum of the numbers in the major diagonal that starts with 1 and moves downward to 49. Find the sum of the numbers in the major diagonal that starts with 43 and moves downward to 7.

6. What is the relationship between the sums of the two major diagonals?

7. Does this relationship exist in the 6 x 6 Number Table?

ACTIVITY THREE: 10 x 10 Table

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
11	**12**	**13**	**14**						

Table 3
10 x 10 Numbers are arranged into 10 rows and 10 columns

1. Complete the table.

2. Let's look at the major diagonal, starting at 1 in the upper left corner and moving to the lower right corner. Predict the relationship based on your discoveries in 6 x 6 Number Table and 7 x 7 Number Table.

3. Let's look at the major diagonal, starting at 91 in the lower left corner and moving to the upper right corner. Predict the relationship based on the past Number Tables.

4. What about the relationships of the minor diagonals? Such as moving downward from 51 to 62 to 73 to 84 to 95? How about moving upward from 61 to 52 to 43 to 34 to 25 to 16 to 7? Do those relationships work with all minor diagonals? How did you know this before you did the calculations?

5. What is the relationship between the sums of the two major diagonals?

6. On the back of this handout, list some other relationships that you find in this 10 x 10 Number Table.

ACTIVITY FOUR: Pascal's Triangle

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U
0											1										
1										1		1									
2									1		2		1								
3								1		3		3		1							
4							1		4		6		4		1						
5						1										1					
6					1												1				
7				1														1			
8			1																1		
9		1																		1	
10	1																				1

Table 4
Pascal's Triangle[1]

1. This pattern continues by adding the 2 numbers above the position you wish to fill. For example on Row 4, 3 is the sum of 1 + 2. In Row 5, 6 is the sum of 3 + 3. Complete the table. Each row starts and ends with a 1.

2. What is the relationship of the numbers moving diagonally from the top 1 to the lower right and ending on the last 1 in the bottom row?

3. What is the relationship starting with the first 1 in the second row and moving diagonally to the lower right?

4. What is the relationship starting with the first 1 in the third row and moving diagonally to the lower right?

5. Do these same relationships occur anywhere else in Pascal's Triangle?

6. What other patterns do you notice? Any patterns with columns or rows?

7. Search the web sites provided by your teacher to find out where and how Pascal's Triangle is used.

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[1] **NOTE:** For reference in the questions, rows are labeled with numbers at the beginning of each row and columns are labeled with capital letters. These labels are not part of Pascal's Triangles.