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Exploring Number Patterns Using Number Boards

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The base 10 system of counting commonly used today uses 10 digits, specifically 1, 2, 3, 4, 5, 6, 7, 8, 9, and 0. Although we do not use zero in counting, it does assign place value to numbers, and thus, is extremely valuable. On the Hundred Board, infinite patterns can be explored to help students develop higher thinking skills. The most fundamental pattern is counting by tens. We should teach patterns to promote students' ability to recognize similarities quickly and correctly and be able to apply patterns to different venues and subjects.

The Hundred Board arranges the first 100 numbers into 10 rows and 10 columns; our assignment is to create number boards for 6 and 13. For each number board, students look for patterns to draw appropriate conjectures. A summary is included to describe the patterns for each of the numbers and to determine whether any of the number patterns are better than the tens' pattern.

Before arranging the sample numbers into patterns, it is helpful to discuss divisibility rules and the definitions of prime and composite numbers. These rules help to determine patterns for prime numbers as well as relationships in other numbers, such as multiples of 6.

To begin, any number greater than 1 that is solely divisible by 1 and the number itself is a *prime number*; a number with more than two divisors is a *composite number*.

Table 1. Divisibility Rules for Divisors to 10

1	All natural numbers {1, 2, 3, …}
2	Last digit is divisible by 2
3	Sum of the digits is divisible by 3
4	Number formed by the last two digits is divisible by 4
5	Last digit is 0 or 5
6	Number is divisible by 2 and 3
8	Number formed by the last three digits is divisible by 8
9	Sum of the digits is divisible by 9
10	Last digit is 0

The Hundred Board arranges the first 100 numbers into 10 rows and 10 columns. Utilizing the Hundred Board as a base, the following number patterns can be discovered!

Patterns with 6

Table 2 shows some of the numbers from the Hundred Board rearranged so there are only 6 numbers in a row.

Table 2. Numbers arranged into 6 rows and 6 columns

1	2	3	4	5	6
7	8	9	10	11	12
13	14	15	16	17	18
19	20	21	22	23	24
25	26	27	28	29	30
31	32	33	34	35	36

With this new arrangement of numbers, we observe the following patterns.

1. The numbers in the last column are all divisible by 6.
2. All numbers in the 2nd, 4th, and 6th columns are even; numbers in the 1st, 3rd, and 5th columns are odd.
3. Numbers are arranged in each column so that when going from one row to the cell in the next row down you count by 6; this is true in all the columns.

Table 3. 6 $\times$ 6 Numbers By Diagonals

1	2	3	4	5	6
7	8	9	10	11	12
13	14	15	16	17	18
19	20	21	22	23	24
25	26	27	28	29	30
31	32	33	34	35	36

4. The diagonals from the top right to bottom left have a pattern of $n + 5$; for example, if $n = 3$, then the next diagonal number is 8. The diagonals going upward from bottom left to top right have a pattern of $n - 5$; if $n = 20$, then the next diagonal number is 15. These patterns are indicated with dark arrows in Table 3. The diagonals going from top left to bottom right have a pattern of $n + 7$; if $n = 2$, the next diagonal number is 9. The diagonals going upward from bottom right to top left have a pattern of $n - 7$; if $n = 18$, the next diagonal number is 11. These patterns are indicated with gray arrows in Table 3. The '$n + 5$' and '$n - 5$' patterns are both dark and the 'n + 7' and 'n – 7' patterns are both gray because they are essentially the same pattern; they

are just approached from different directions. One can find a number of different patterns from the same set of data.

5. Numbers at the opposite ends of diagonals all add to 37. As seen in Table 3, **1 + 36 = 37** and **6 + 31 = 37.**

Patterns with 13

Table 4 shows the numbers from the Hundred Board rearranged so that 13 numbers are in a row.

Table 4. Numbers Arranged into 13 rows and 13 columns

1	2	3	4	5	6	7	8	9	10	11	12	13
14	15	16	17	18	19	20	21	22	23	24	25	26
27	28	29	30	31	32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47	48	49	50	51	52
53	54	55	56	57	58	59	60	61	62	63	64	65
66	67	68	69	70	71	72	73	74	75	76	77	78
79	80	81	82	83	84	85	86	87	88	89	90	91
92	93	94	95	96	97	98	99	100	101	102	103	104
105	106	107	108	109	110	111	112	113	114	115	116	117
118	119	120	121	122	123	124	125	126	127	128	129	130
131	132	133	134	135	136	137	138	139	140	141	142	143
144	145	146	147	148	149	150	151	152	153	154	155	156
157	158	159	160	161	162	163	164	165	166	167	168	169

In this table, we observe the following patterns.

1. The numbers in the last column are all divisible by 13.
2. Numbers are arranged in each column so that when going from one row to the cell in the next row down you count by 13; this is true in all the columns.
3. The diagonals going from top right to bottom left have a pattern of $n + 12$; if $n = 30$, the next diagonal number is 42. The diagonals upward from bottom left to top right have a pattern of $n – 12$; if $n = 28$, the next diagonal number is 16. These patterns are indicated with dark arrows in Table 5. The diagonals from top left to bottom right have a pattern of $n + 14$; if $n = 23$, the next diagonal number is 37. The diagonals going upward from bottom right to top left have a pattern of $n – 14$; if $n = 63$, the next diagonal number is 49. These patterns are indicated with gray arrows in Table 5.
4. Numbers at the opposite ends of diagonals add to 170. As seen in Table 5, **169 + 1 = 170**, or **157 + 13 = 170**.
5. In a single column, the rows alternate between even and odd. If the first row starts with an odd number, it ends with an odd number. If the first row starts with an even number, then it ends with an even number.
6. Prime numbers fall on selected diagonals, such as 5, 17, 29, 41, and 53.

Table 5. 13 × 13 Numbers By Diagonals

1	2	3	4	5	6	7	8	9	10	11	12	13
14	15	16	17	18	19	20	21	22	23	24	25	26
27	28	29	30	31	32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47	48	49	50	51	52
53	54	55	56	57	58	59	60	61	62	63	64	65
66	67	68	69	70	71	72	73	74	75	76	77	78
79	80	81	82	83	84	85	86	87	88	89	90	91
92	93	94	95	96	97	98	99	100	101	102	103	104
105	106	107	108	109	110	111	112	113	114	115	116	117
118	119	120	121	122	123	124	125	126	127	128	129	130
131	132	133	134	135	136	137	138	139	140	141	142	143
144	145	146	147	148	149	150	151	152	153	154	155	156
157	158	159	160	161	162	163	164	165	166	167	168	169

Generalizing Patterns

As one looks closer at the patterns that exist within different number arrangements, it becomes obvious that the patterns are constant among them.

1. In the different arrangements, the last column is always divisible by the number pattern being explored. In the 6 x 6 pattern, the numbers in the last column are all divisible by 6; likewise, in the 13 × 13 pattern, the numbers in the last column are divisible by 13.
2. In each column, the numbers increase in value when moving down from one row to the next by the amount for the pattern being explored. In Tables 2 and 4, we counted by 6 or 13, respectively, as we moved from one row to another in the same column.
3. The number arrangements in the diagonals are similar, namely they are either $a \pm (k + 1)$ or $a \pm (k - 1)$ where k = the number pattern being explored (e.g., 6 or 13) and a = any cell in the number pattern. For example, in the 13 × 13 pattern, $k = 13$, and "a" equals any number in the 13 × 13 arrangement.
4. The rows alternate between even or odd. If the column starts with an odd number, it ends with an odd number and vice versa.

Extension of Patterns: Prime Numbers and Multiples of 6

Consider the prime numbers (2, 3, 5,…) and how they are related to multiples of six. Prime numbers greater than or equal to 5 are of the form $6n + 1$ or $6n - 1$ for integers n. Although not all numbers of the form $6n \pm 1$ are prime, *all prime numbers* greater than or equal to 5 *do* follow the $6n \pm 1$ form. Furthermore, when looking at the 6 × 6 pattern, prime numbers, with the exception of 2 and 3, fall into columns 1 or 5 as shown in Table 6.

Table 6. 6×6 Arrangement to Show Prime Numbers

1	**2**	**3**	**4**	**5**	**6**
7	**8**	**9**	**10**	**11**	**12**
13	**14**	**15**	**16**	**17**	**18**
19	**20**	**21**	**22**	**23**	**24**
25	**26**	**27**	**28**	**29**	**30**
31	**32**	**33**	**34**	**35**	**36**

Conclusion

Young children count; they count by 1s, then by 2s, 5s, and 10s. Patterns give students a natural strategy to understand addition and multiplication. When considering a number pattern such as 6 or 13, a young student will ask himself, "By what number can I count (add) to get to the next number in the pattern and the next and the next?"

Many commonalities exist among the different patterns. It can be good for students to get out of their comfort zone of base 10 number arrangements. Different number patterns may intrigue a student's sense of discovery and serve as a teaching tool.

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